Unconventional anomalous Hall effect from antiferromagnetic domain walls of Nd$_2$Ir$_2$O$_7$ thin films

Woo Jin Kim,1,2 John H. Gruenewald,3 Taekoo Oh,1,2,4 Sangmo Cheon,5 Bongju Kim,1,2 Oleksandr B. Korneta,1,2 Hwanbeom Cho,1,2 Daesu Lee,1,2 Yoonkoo Kim,6 Miyoung Kim,6 Je-Geun Park,1,2 Bohm-Jung Yang,1,2,4 Ambrose Seo,3 and Tae Won Noh1,2,*

1Center for Correlated Electron Systems, Institute for Basic Science, Seoul 08826, Republic of Korea
2Department of Physics and Astronomy, Seoul National University, Seoul 08826, Republic of Korea
3Department of Physics and Astronomy, University of Kentucky, Lexington, Kentucky 40506, USA
4Center for Theoretical Physics, Seoul National University, Seoul 08826, Republic of Korea
5Department of Physics, Hanyang University, Seoul 04763, Republic of Korea
6Department of Materials Science and Engineering and Research Institute of Advanced Materials, Seoul National University, Seoul 08826, Republic of Korea

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Ferroic domain walls (DWs) create different symmetries and ordered states compared with those in single-domain bulk materials. In particular, the DWs of an antiferromagnet with noncoplanar spin structure have a distinct symmetry that cannot be realized in those of their ferromagnet counterparts. In this paper, we show that an unconventional anomalous Hall effect (AHE) can arise from the DWs of a noncoplanar antiferromagnet, Nd$_2$Ir$_2$O$_7$. Bulk Nd$_2$Ir$_2$O$_7$ has a cubic symmetry; thus, its Hall signal should be zero without an applied magnetic field. The DWs generated in this material break the twofold rotational symmetry, which allows for finite anomalous Hall conductivity. A strong $f$-$d$ exchange interaction between the Nd and Ir magnetic moments significantly influences antiferromagnetic (AFM) domain switching. Our epitaxial Nd$_2$Ir$_2$O$_7$ thin film showed a large enhancement of the AHE signal when the AFM domains switched, indicating that the AHE is mainly due to DWs. Our paper highlights the symmetry-broken interface of AFM materials as a means of exploring topological effects and their relevant applications.

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I. INTRODUCTION

Both symmetry and the ferroic order inevitably become modified in a ferroic domain wall (DW), triggering new functions and topological properties [1–6]. The discovery of conductive DWs in ferroelectric thin films opened a broad field of research dedicated to understanding the underlying mechanisms, and the fabrication of practical DW-based devices [1–4]. The DWs of certain ferromagnets exhibit topological Hall effects attributable to magnetic skyrmions [5,6]. However, similar topological properties have yet to be observed in the DWs of antiferromagnets. Although there exists a theoretical prediction that a finite anomalous Hall effect (AHE) can arise among antiferromagnetic (AFM) DWs with nonvanishing Berry curvatures [7], few experimental studies have investigated this intriguing possibility.

Since Wan et al. theoretically proposed the existence of a Weyl fermionic state in AFM-ordered Y$_2$Ir$_2$O$_7$ [8], the rare-earth pyrochlore iridates have attracted great interest [9]. Particularly, Nd$_2$Ir$_2$O$_7$ has been widely studied, due to its proximity to the metal-insulator transition (MIT) [10,11] and large DW conductance [12]. To discuss the AFM spin structure in detail, consider that Nd$_2$Ir$_2$O$_7$ is composed of two types of tetrahedra, Ir and Nd sublattices, as shown in Fig. 1(a). As shown by the red arrows, all four of the Ir spins at the vertices of a tetrahedron point inward. At the nearest-neighbor Ir tetrahedra (not shown), all of the Ir spins should point outward. This results in all-in–all-out (AIAO) ordering of Ir spins, which we refer to as Ir:AIAO. At the same time, the spins in the Nd sublattice have similar magnetic ordering (all-out–all-in), Nd:AOAI, as shown in Fig. 1(a). These intriguing magnetic orderings and coupling between two different sublattices play important roles in the physical properties of these materials [8–14].

Under an external magnetic field, $H$, we can switch between AIAO and AOAI domains of Ir spin. It is important to understand how DWs can be formed during the switching process in a Nd$_2$Ir$_2$O$_7$ sample. Note that the Nd ion has a larger magnetic moment; thus, the corresponding Zeeman field is larger. When $H$ increases along the [111] direction, Nd spins become canted and then suddenly flip to form the three-in–one-out (3I1O) configuration [15]. At a higher $H$, Nd spins can induce a flip of the Ir sublattice spin into the AOAI configuration via the $f$-$d$ exchange interaction [see Fig. 1(b)]. During this spin switching process, two kinds of domains may coexist, resulting in the presence of DWs [see Fig. 1(c)].

Here, we present a Nd$_2$Ir$_2$O$_7$ epitaxial thin film as an ideal system for realizing the intriguing topological responses that originate from the DWs of an AFM material. The electronic structure of Nd$_2$Ir$_2$O$_7$ has been assumed to be close to that of a Weyl semimetal [8,16]. This similarity to a semimetallic ground state results in a very large DW conductance [12,15]. Within a single AIAO or AOAI domain, AHE is forbidden.

*twnoh@snu.ac.kr
FIG. 1. Schematic diagrams of the magnetic domain structure of Nd$_2$Ir$_2$O$_7$. (a) The magnetic structure of Nd 4$f$ moments (blue arrows) and Ir 5$d$ moments (red arrows) at $H = 0$ and temperatures below $T_N^{Nd}$. The Ir and Nd sublattices have all-in–all-out (AIAO) and all-out–all-in (AOAI) ordering, respectively. (b) The magnetic structure in sufficiently strong $H$. The Nd 4$f$ and Ir 5$d$ moments are coupled with the $f$–$d$ exchange interaction. (c) Schematic diagram of the DWs between Ir-AIAO and Ir-AOAI. The purple-colored line indicates the possible DW structure with finite magnetization. (d) Schematic diagram of the dotted square region in (c), enlarged for detail. The purple arrows indicate the net magnetization of Ir moments at a DW.

by cubic crystalline symmetry [17,18]. However, in the presence of a DW, we show that twofold rotation symmetries are explicitly broken, resulting in uncompensated magnetic moments (see Sec. IV C). This leads to a finite anomalous Hall conductivity in the plane parallel to the highly conducting DW. Thus, it is informative to explore DW-induced unconventional magnetotransport in low-dimensional Nd$_2$Ir$_2$O$_7$ thin films, in which the contribution of the DW can be maximized [19].

II. EXPERIMENTS

A. Deposition of Nd$_2$Ir$_2$O$_7$ thin films

We prepare Nd$_2$Ir$_2$O$_7$ (111) thin films on commercial Y-stabilized ZrO$_2$ (YSZ) (111) single-crystal substrates via pulsed laser deposition followed by a postannealing procedure. We irradiate a single-phase Nd$_2$Ir$_2$O$_7$ polycrystalline target with a KrF excimer laser ($\lambda = 248$ nm); the laser fluence and frequency were 4.5 J/cm$^2$ and 3 Hz, respectively. We maintain the distance between the target and the substrate at 50 mm. It is well known that pyrochlore iridate thin films under such conditions, the gas phase of IrO$_3$ is likely to form and evaporate, posing extreme difficulties for \textit{in situ} growth. We initially deposit stoichiometric amorphous Nd$_2$Ir$_2$O$_7$ thin films at 600 °C. Following deposition, we postanneal the thin films in air in an electrical box furnace at 1000 °C for 1 h [24].

Figure 2(a) shows x-ray-diffraction (XRD) $\theta$-2$\theta$ scans of our Nd$_2$Ir$_2$O$_7$ thin film and the YSZ substrate. The figure shows only peaks from plane parallel to Nd$_2$Ir$_2$O$_7$ (111) in addition to substrate peaks, indicating that the thin film is a single phase. Figure 2(b) shows the detailed XRD pattern near the Nd$_2$Ir$_2$O$_7$ (222) peak. Figure 2(c) shows that the full width at half maximum of the rocking curve is $\sim 0.05^\circ$, indicating that the sample is highly crystalline. We utilize x-ray reciprocal space mapping (X-RSM) to measure the in-plane lattice constants of the thin film and substrate. Figure 2(d) shows X-RSM data around the (662) and (331) Bragg reflections of the Nd$_2$Ir$_2$O$_7$ thin film and YSZ substrate, respectively. The (662) Bragg peak of the Nd$_2$Ir$_2$O$_7$ thin film has smaller $Q_x$ and $Q_z$ values than those of the YSZ (331) Bragg peak. From the experimental $Q_x$ and $Q_z$ values, the lattice constants of our thin film are estimated to be $a = b = 10.380$ Å, close to those of bulk polycrystalline Nd$_2$Ir$_2$O$_7$ ($a = b = c = 10.375$ Å). This indicates that our thin films have high structural quality close to the bulk counterparts.

B. Scanning transmission electron microscopy measurements

We utilize scanning transmission electron microscopy (STEM) to explore the microstructure of our Nd$_2$Ir$_2$O$_7$ thin films. Figure 3(a) shows a STEM image of an Nd$_2$Ir$_2$O$_7$ thin
FIG. 3. (a) STEM image of an Nd$_2$Ir$_2$O$_7$ thin film deposited on YSZ (111) from the viewpoint of the [2\bar{1}\bar{1}] axis. The Nd$_2$Ir$_2$O$_7$ thin film exhibits an ordered pyrochlore structure and an epitaxial relationship with the YSZ substrate. (b) The fast Fourier transform of the pyrochlore Nd$_2$Ir$_2$O$_7$ thin film (upper red dashed square) and (c) that of the YSZ substrate (lower red dashed square).

A blurry region is evident near the interface between thin film and substrate, which may be attributable to defects and/or electron-beam damage during STEM measurements. To determine structural relationships, we perform fast Fourier transforms (FFTs) of real-space images of the Nd$_2$Ir$_2$O$_7$ thin film and the YSZ substrate region [see Figs. 3(b) and 3(c)]. Although our thin films are deposited by postannealing the amorphous phase, an epitaxial relationship is evident between Nd$_2$Ir$_2$O$_7$ and the YSZ substrate: Nd$_2$Ir$_2$O$_7$ [111] $\parallel$ YSZ [111] and Nd$_2$Ir$_2$O$_7$ [0\bar{1}1] $\parallel$ YSZ [0\bar{1}1].

C. Resistivity and magnetotransport measurements

We measure dc magnetotransport properties below $\sim$14 T using the standard four-probe method and a commercial cryostat system (Oxford Instruments). For higher H fields up to 30 T, we utilize the resistive magnet at the National High Magnetic Field Laboratory. During these magnetotransport measurements, we apply current along the [1\bar{1}0] direction and H along the [111] direction (i.e., perpendicular to the current direction). In Fig. 4(a), our 80-nm-thick Nd$_2$Ir$_2$O$_7$ thin film exhibits a MIT around $T_{Ir}^N \sim$ 30 K, close to the bulk value. On the other hand, the associated resistivity ratio, $\rho(2\text{K})/\rho(300\text{K})$, is much broader than that of the “best” single-crystalline sample [15]. The resistivity ratio, $\rho(2\text{K})/\rho(300\text{K})$, is $\sim$10, which is much smaller than the compared value ($\sim$1000) of the single crystal. However, it should be noted that the reported resistivity ratio values in literature vary in a wide range of 10–1000 even for single crystals [25]. Therefore, our Nd$_2$Ir$_2$O$_7$ thin films has crystal quality at least comparable to some single crystals [25].

To measure magnetoresistance (MR) behavior, we zero-field cooled the Nd$_2$Ir$_2$O$_7$ thin film to 2 K and measured resistivity under a constant H field while warming to 300 K. In Fig. 4(b), the resistance changes under a H field are large but become smaller as T increases, vanishing around $T_{Ir}^N$. Figure 4(c) shows T-dependent normalized MR curves at various H values. The MR peaks at $\sim$15 K, which corresponds to the Néel ordering temperature of the Nd moment ($T_{Nd}^N$) [26].

III. ANTIFERROMAGNETIC DOMAIN SWITCHING

A. Magnetoresistance hysteresis

Figure 4(d) shows a H-dependent normalized MR curve at 2 K, where the comparison with the single crystal data in Ref. [15] can be made. Note that it exhibits an intriguing hysteretic behavior with very broad MR dips around $\pm 3$ T. This feature is somewhat broad, since it may come from many inherent defects inside the thin film and pinning of DWs to such defects. However, this hysteretic behavior is essentially similar to that of the single crystal, which is marked by the blue square line. The MR dips occurs around $\pm 3$ T, nearly at the magnetic coercive field $H_C$ of the single crystal [15]. In addition, the hysteresis direction is the same as that in
and AOAI domains. Data can be explained in terms of switching between the AIAO and asymmetric part of the normalized MR curves using attributed to AFM domain switching [27]. Likewise, our MR quantified fit parameters Eq. (2) and its total contribution to both Nd and Ir, respectively. (c) Lines indicate the hysteretic part of Ir (Nd) domain switching from the represented spin structures. The red (blue) and orange solid (positive [111]. (b) Asymmetric component of the sweep-up MR at 1.8 K formation on magnetic domain switching [24,28]. We extract known that the asymmetric MR part contains important in-

terfaces below $T_N$ [26], consistent with Fig. 4(c). Note that the dip in $T_{N}^{\text{Ir}}$ is plotted, given that $\text{MR}_{\text{Asym}}(-H) = -\text{MR}_{\text{Asym}}(H)$. Note the interesting structures in the $\text{MR}_{\text{Asym}}(H)$ curve: a peak at low $H$ and a dip at high $H$. Additionally, the curve displays a sign change in $\text{MR}_{\text{Asym}}(H)$ around 3.0 T. To our knowledge these $\text{MR}_{\text{Asym}}(H)$ behaviors have not been observed in other magnetic ferroic materials.

Using detailed analysis of the MR curves, we can obtain insight into spin-ordering and DW dynamics. It is well established in most ferroic materials that domain switching usually occurs at $H_C$. When switching becomes broadened due to pinning of DWs to defects and/or surfaces, the $H_C$ distribution in DW switching dynamics can be fitted with a Gaussian function [29,30]. To account for the asymmetry, we fit our $\text{MR}_{\text{Asym}}(H)$ data with an asymmetric Gaussian function:

$$\text{MR}_{\text{Asym, FFT}}(H) = a e^{-b(\text{a} H + \text{b} H^2)}$$

where $a$ and $b$ are the amplitude and width of the hysteretic behavior, respectively.

We find that the associated sign change in $\text{MR}_{\text{Asym}}(H)$ in Fig. 5(b) cannot be fit with a single asymmetric Gaussian function in Eq. (2). This indicates that the hysteresis cannot be explained in terms of magnetic switching in one kind of sublattice, i.e., either Ir or Nd spins. On the other hand, when we introduce two magnetic switching functions (namely, the two asymmetric Gaussian functions), we are able to fit the experimental $\text{MR}_{\text{Asym}}(H)$ curves quite well. In Fig. 5(b), we plot the red (blue) line for higher-field (lower-field) magnetic switching. These indicate that both Ir and Nd spins are involved with magnetotransport properties of our Nd$_2$Ir$_2$O$_7$ thin films.

Considering the $T_N$ of each ion, we find that the switching at the higher (lower) $H_C$ value is the result of the Ir (Nd) sublattice. Figure 5(c) shows the temperature dependence of each respective ion’s fit coefficient ($a$) calculated from the symmetry analysis of $\text{MR}_{\text{Asym}}(H)$ curves. As the temperature is decreased, the $a$ value of the red line first appears around 30 K ($\sim T_N^{\text{Ir}}$). This coincides with Ir spin ordering; thus, we can assign the red line to $a^{\text{Ir}}$. Likewise, the blue line begins to emerge around $T_N^{\text{Nd}}$ and monotonically increases, thus corresponding to $a^{\text{Nd}}$. Note that the values of $a^{\text{Ir}}$ are larger than those of $a^{\text{Nd}}$. As the conduction in Nd$_2$Ir$_2$O$_7$ occurs in the Ir-O network [9], it is reasonable to expect the Ir spins to dominate the change in MR. Therefore, the red and blue lines in Fig. 5(b) should correspond to the switching of spins of Ir and Nd sublattices, respectively.

C. Domain switching caused by the $f-d$ exchange interaction

Figure 5(b) shows that the switching of Nd spins occurs at lower $H$ than that of Ir spins. Based on the relative values of the switching $H$ field, we can summarize the domain switching sequence as follows: (1) at small $H$, the Ir-AIOA domains dominate, along with Nd-AOAI spin structures; (2) as $H$ increases, Nd spins flip first to form Nd-3I1O while the Ir sublattice remains in the Ir-AIOA configuration; and (3) finally, at higher $H$, the Nd-3I1O spin structure flips the Ir spins, resulting in the formation of Ir-AOAI domains. This switching sequence is consistent with that described in Fig. 1.

The MR data also indicate that Ir and Nd spins are strongly coupled, presumably due to the $f-d$ exchange interaction.
From the $T$-dependent analysis of MR$_{\text{Asym}}(H)$, we are able to determine the coercive fields required to switch the spins of each Ir and Nd sublattice. The fitted values for $\mu_0 H_{C}^{\text{Nd}}$ are shown as blue open diamonds in Fig. 5(d). Nd spin switching does not occur for $T > T_{\text{N}}^{\text{Nd}}$. Below $T_{\text{N}}^{\text{Nd}}$, $\mu_0 H_{C}^{\text{Nd}}$ increases and remains nearly constant around 2 T. The fitted values for $\mu_0 H_{C}^{\text{Ir}}$ are shown as red open circles. Ir-AIAO to AOAI domain switching begins below $T_{\text{N}}^{\text{Ir}}$, i.e., $H_{C}^{\text{Ir}} \sim 7$ T around 20 K. The MR signal becomes small close to $T_{\text{N}}^{\text{Ir}}$, making it difficult to estimate a value for $H_{C}^{\text{Ir}}$. As the temperature decreases below 30 K, hysteretic behavior with a “humplike” peak is observed, which cannot be explained by the conventional ordinary Hall effect. This hysteretic behavior emphasizes the important role of the $J-d$ exchange interaction in Ir domain switching of the pyrochlore iridate thin film.

IV. ANOMALOUS HALL EFFECTS CAUSED BY ANTIferROMAGNETIC DOMAIN WALLS

A. AHEs observed in Nd$_2$Ir$_2$O$_7$ thin films

We measure the transverse magnetoresistance, i.e., Hall resistivity $\rho_{xy}$, of our Nd$_2$Ir$_2$O$_7$ thin films with $H$ along the [111] direction. Figure 6(a) shows that the $\rho_{xy}$ curves have a very large and unconventional Hall resistivity behavior. As the temperature decreases below 30 K, hysteretic behavior with a “humplike” peak is observed, which cannot be explained by the conventional ordinary Hall effect. This hysteretic behavior with humplike peak intensity becomes more pronounced as the temperature decreases below $T_{\text{N}}^{\text{Nd}}$. Similar humplike behavior has been observed in ferromagnetic DWs and has been attributed to the presence of a real-space topological Hall effect related to magnetic skyrmions [5,6].

To obtain further insight into the AHE, we subtracted the $H$-linear ordinary Hall effect term from the 2-K $\rho_{xy}$ data and plotted the data as black solid circles in Fig. 6(b). In the figure, the AHE term peaks around 2–3 T. The corresponding $H$ value is close to the $\mu_0 H_{C}^{\text{Nd}}$ values, obtained from the earlier MR data analysis. It indicates that a DW could play a significant role in the observed AHE. In addition, the experimental AHE reaches a nearly constant value above 5 T, suggesting that bulk effects may also be involved. These results suggest that the AHE may have originated from both bulk and DW contributions, hereafter referred to as bulk AHE and DW AHE, respectively.

B. Bulk AHE of antiferromagnetic material: Scalar spin chirality

For AFM bulk materials, AHEs have been observed and explained in terms of the scalar spin chirality. This term can be described as $\vec{S}_i \cdot (\vec{S}_j \times \vec{S}_k)$ with three local spins $\vec{S}_i$, $\vec{S}_j$, and $\vec{S}_k$ [31]; the scalar product corresponds to the solid angle $\Omega$ subtended by the three spins. The noncoplanar spin configuration is closely linked to the Berry curvature and can generate a fictitious magnetic flux proportional to the scalar spin chirality. As a result, an AHE can occur [32–39]. Without a magnetic field, the total magnetization and scalar spin chirality remain zero inside a single Nd$_2$Ir$_2$O$_7$ magnetic domain. However, applying a strong magnetic field changes the order of the Nd moments to 3I1O [see Fig. 1(b)] and Ir moments to a canted AIAO. Then, both magnetization and scalar spin chirality become finite, and the canted antiferromagnet exhibits finite bulk AHE.

Let us estimate the $\rho_{xy}$ values attributable to the bulk AHE for Nd$_2$Ir$_2$O$_7$. Note that the AIAO (AOAI) order of the Ir moments is canted by the 1I3O (3I1O) order of the Nd moments via the $J-d$ exchange interaction. As a result, the scalar spin chirality is produced in the Ir spin system and generates a fictitious magnetic field parallel to its magnetization direction [40]. Moreover, it is well known that the Nd moments ($\sim 2.4 \mu_B$/Nd) are much larger than the Ir moments ($\sim 0.2 \mu_B$/Nd) [15]. As the magnetic moment of Ir is an order of magnitude smaller than that of Nd, the Ir sublattice makes a much smaller contribution to the magnetization. Given that both the scalar spin chirality term of Ir spins and total magnetization are dominated by Nd moments, we will focus on the spin configuration in Nd tetrahedrons, changing from AIAO to 3I1O. We can quantify the proportions of Nd unit cells in the 3I1O order by calculating the expectation value of magnetization per unit cell. Then we can infer a bulk Hall resistivity curve, which is proportional to the number of Nd unit cells in the 3I1O order.

To quantitatively estimate the $H$-dependent net magnetization of the Nd sublattice, we consider the Hamiltonian below, which describes a single Nd unit cell in a magnetic field along...
the direction of the [111] plane:

\[ H = -J \sum_{(ij)} \mathbf{S}_i \cdot \mathbf{S}_j - \mu \mathbf{B} \cdot \sum_i \mathbf{S}_i - K \Phi B_x B_y B_z, \]  

where \( J \) is the interaction strength, \( \mu \) is the Bohr magneton, and \( \mathbf{B} \) is magnetic field. \( K \) is a coefficient and \( \Phi \) is the AIAO order parameter expressed as

\[ \Phi = \frac{1}{4\sqrt{3}} (S_{1x} + S_{1y} + S_{1z} + S_{2x} - S_{2y} - S_{3x} + S_{3y} - S_{3z} - S_{4x} - S_{4y} + S_{4z}). \]  

(4)

Here, we treat the Nd spin as an Ising variable. The spin vector at each site in a unit cell is defined as \( \mathbf{S}_1 = \frac{S}{\sqrt{3}}(1, 1, 1), \mathbf{S}_2 = \frac{S}{\sqrt{3}}(1, -1, 1), \mathbf{S}_3 = \frac{S}{\sqrt{3}}(-1, 1, -1), \) and \( \mathbf{S}_4 = \frac{S}{\sqrt{3}}(-1, -1, 1), \) where \( S_i = \pm 1. \) Then, Eq. (3) can be simplified to

\[ H = -J \sum_{(ij)} \sigma_i \sigma_j - \mu \mathbf{B} \cdot \sum_i \mathbf{S}_i - K \Phi B_x B_y B_z. \]  

(5)

When \( J > 0, \) the ground state of the Hamiltonian in a Nd unit cell is AIAO or AOAI ordered without the magnetic field; namely, \( \sigma_i \) is either +1 or −1. As such, the expectation value of the magnetization can be calculated using a partition function. For the unit cell, the partition function is

\[ Z = \sum_{\sigma_1, \sigma_2, \sigma_3, \sigma_4 = \pm 1} \exp(-\beta H), \]  

(6)

and the expectation value of magnetization along the [111] direction is

\[ \langle M \rangle = \frac{1}{Z} \sum_{\sigma_1, \sigma_2, \sigma_3, \sigma_4 = \pm 1} \left( \sigma_1 - \frac{1}{3} (\sigma_2 + \sigma_3 + \sigma_4) \right) \exp(-\beta H). \]  

(7)

We evaluated the bulk AHE of our Nd$_2$Ir$_2$O$_7$ thin film by assuming that it should be proportional to the magnetization. Given a sufficient \( H \) field, all of the DWs should disappear, such that the DW AHE becomes negligible and the bulk AHE dominates. Using the experimental values of the saturated Hall resistivity value (≈8 \( \mu \)Ω cm) at \( \sim 2 \) T (\( \mu \)0H$_{\text{Nd}}$) in Fig. 6(b), we derived an analytical estimate of the bulk AHE contribution. The bulk-AHE contribution is shown as the blue line in Fig. 6(b). It should be noted that the bulk AHE alone cannot explain the experimental AHE. There should be an additional AHE, which cannot be explained by the scalar spin chirality effect.

C. Emergence of the AHE in the presence of an AFM domain wall: Symmetry analysis

We considered the possible emergence of the AHE due to momentum space Berry curvature by performing symmetry analysis. First, we should note that the anomalous Hall conductivity cannot occur in a single domain of AIAO order in cubic pyrochlore iridates [17]. To calculate the intrinsic anomalous Hall conductivity, we integrate the Berry curvature of the occupied bands in momentum space:

\[ \sigma_{\alpha\beta} = \frac{e^2}{h} \int d^3 \mathbf{k} \sum_n f(\epsilon_n(\mathbf{k}) - \mu) F_{\alpha\beta}(\mathbf{k}), \]  

(8)

where \( f(\epsilon_n(\mathbf{k}) - \mu) \) is the Fermi-Dirac distribution, and \( F_{\alpha\beta} \) is the Berry curvature of the \( \alpha\beta \) plane. For convenience, we denote each component of the Berry curvature in a vector notation, i.e., \( F^x(\mathbf{k}) = F_{yx}(\mathbf{k}), F^y(\mathbf{k}) = F_{xz}(\mathbf{k}), \) and \( F^z(\mathbf{k}) = F_{xy}(\mathbf{k}). \) When twofold rotation symmetry around the \( z \) axis (\( C_{2z} \)) exists, as in the single domain of AIAO (or AOAI) order, \( F^x(-k_x, -k_y, k_z) = -F^x(k_x, k_y, k_z), F^y(-k_x, -k_y, k_z) = -F^y(k_x, k_y, k_z), \) and \( F^z(-k_x, -k_y, k_z) = F^z(k_x, k_y, k_z). \) Then \( \sigma_{zx}, \sigma_{zy} \) should be trivial, but \( \sigma_{xy} \) does not have to be trivial through Eq. (8). In a single domain of AIAO order, cubic pyrochlore iridates have \( C_{2x}, C_{2y}, \) and \( C_{2z} \); symmetry, thus, AHEs cannot occur.

However, in the presence of a DW, an unconventional AHE can be generated from the nonzero Berry curvature. At the DW, twofold rotation symmetries become broken but there exists a threefold rotational symmetry about the axis perpendicular to the DW (see Fig. 7). Considering the threefold rotation axis as [111] and taking the two other perpendicular axes as [011] and [211], the symmetry properties of the Berry curvature under threefold rotational symmetry are \( F^x(k_x, k_y, k_z) = F^x(k_y, k_z, k_x), F^y(k_x, k_y, k_z) = F^y(k_z, k_x, k_y), \) and \( F^z(k_x, k_y, k_z) = F^z(k_x, k_z, k_y). \) Using these properties, we establish the properties of the Berry curvature components along [111] as

\[ F^{[111]}(\mathbf{k}) + F^{[111]}(C\mathbf{k}) + F^{[111]}(C^{-1}\mathbf{k}) \neq 0, \]  

(9)

where \( C \) is the threefold rotation operator. Therefore, \( \sigma^{[111]} \neq 0, \) and finite anomalous Hall conductivity can occur in the plane parallel to the DW.

D. Unconventional AHE at the antiferromagnetic domain walls of Nd$_2$Ir$_2$O$_7$ thin films

When \( H \) is applied parallel to the [111] direction, DWs are expected to form within the [111] plane. Among the possible
DW orientations (see Appendix A), in a zero \( H \) field, the AIAO magnetic ground state favors the formation of DWs within the (111) plane due to its lower frustration compared with other possible orientations [41,42]. Moreover, a magnetization experiment on a \( \text{Cd}_2\text{Os}_2\text{O}_7 \) single crystal, another AIAO ordered pyrochlore structure, showed that the plane of DW formation prefers to orient normal to the direction of the applied \( H \) [42]. Thus, in our experimental geometry, DWs with the (111) plane orientation should play a significant role in magnetotransport results.

Figure 6(b) shows the \( H \)-dependent \( \rho_{xy} \), at 2 K, from which we subtract the ordinary term that varies linearly with \( H \). As mentioned earlier, we should interpret this unconventional Hall resistivity as a combination of the bulk AHE and the DW AHE. The bulk AHE is shown as the blue curve, estimated by following the procedure in Sec. IV B. To obtain the Hall resistivity attributable to DW AHE, we have subtracted the bulk AHE from the experimental values. The red circles in Fig. 6(b) show the remaining contribution.

Note that the maximum value of the remaining Hall resistivity \( \Delta \rho_{xy} \sim 15 \mu \Omega \text{cm} \) is twice as large as the saturated Hall resistivity value of \( 8 \mu \Omega \text{cm} \). The obtained maximum value of the red circles (\( \sim 2 \text{T} \)) is also in reasonable agreement with \( \mu_0 H_{tr}^B (\sim 3 \text{T}) \) from MR where a DW conductance contribution played a significant role. The red circles follow the Gaussian functions used to fit MR\(_{sym, \text{HTT}}(H) \) [see Eq. (2)]. This implies that broad DW switching occurs upon an applied \( H \), which is consistent with our MR analysis. Moreover, the quadratic dependence of the DW anomalous Hall resistivity on longitudinal resistivity indicates that intrinsic AHEs arose at \( \mu_0 H_{tr}^B (\sim 3 \text{T}) \) (see Appendix B). Thus, our results indicate that the humplike \( \rho_{xy} \) signal originates from the DW AHE.

Figure 6(c) shows a contour plot of the \( \rho_{xy} \), from which we subtract the ordinary term, as a function of both \( T \) and \( H \). The contour plot also supports a large enhancement of AHE due to DWs. For a given \( T \), the \( H \) value at which \( \rho_{xy} \) reaches its maximum is correlated with the value of \( H_{tr}^B \) from MR. This implies that the large density of DWs near \( H_{tr}^B \) can result in a maximized contribution to the AHE. The peak value of the humplike Hall resistivity is also highly enhanced below \( T_N \text{Nd} \), which supports the idea of a strong coupling between Ir and Nd sublattices via the \( f-d \) exchange interaction.

It has already been observed experimentally that the DWs of \( \text{Nd}_2\text{Ir}_2\text{O}_7 \) have a much higher conductivity than bulk \( \text{Nd}_2\text{Ir}_2\text{O}_7 \). Their conductivity is approximately one order of magnitude larger [15,43]. Hence, the DW AHE may be significantly enhanced in \( \text{Nd}_2\text{Ir}_2\text{O}_7 \) thin films. In comparison, Hall data on \( \text{Sm}_2\text{Ir}_2\text{O}_7 \) thin films, in which the DW conductance is lower than that of the bulk [44], do not reveal any hysteretic AHE, as shown in Fig. 10 in Appendix C. This observation also indirectly supports that the hysteretic humplike AHE observed in \( \text{Nd}_2\text{Ir}_2\text{O}_7 \) thin films originates from DW conduction.

V. SUMMARY

In summary, we observe a large AHE in an AFM \( \text{Nd}_2\text{Ir}_2\text{O}_7 \) thin film, which is induced mainly by the DWs. The strong \( f-d \) exchange interaction effectively lowers the energy barrier for Ir domain switching, such that a small magnetic field can be used to control the Ir-DW. Eventually, effective Ir-domain switching induces an intrinsic AHE from the Berry curvature at the DW leading to a large DW AHE, \( \sim 15 \mu \Omega \text{cm} \) at 2 K. Given the observation of AHE at DWs, we suggest that \( \text{Nd}_2\text{Ir}_2\text{O}_7 \) is a fertile area for the discovery and investigation of new topological phenomena. As the DW-conducting channels of \( \text{Nd}_2\text{Ir}_2\text{O}_7 \) are attributable to the fact that the system lies close to the topological Weyl semimetal phase, the observed DW-driven AHE provides additional evidence that the system exhibits important topological properties.

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APPENDIX A: ORIENTATION OF THE MAGNETIC DOMAIN-WALL PLANE

The local structures of DWs are considered in terms of the classical spin model. The stability of DWs has been studied in the extreme limit of the strong Ising anisotropy [42]. Three types of spin configurations are possible at local structures of DWs: Ir-AIAO or Ir-AOAI, Ir-3I3O or Ir-113O, and Ir-2I2O. The energies of these configurations are given as \( E_{\text{AIAO}} = -6 J_{\text{eff}}, E_{\text{AOAI}} = 0 \), and \( E_{\text{2I2O}} = 2 J_{\text{eff}} \), where \( J_{\text{eff}} \) is the nearest-neighbor effective antiferromagnetic interaction (\( J_{\text{eff}} > 0 \)). The stability between these DWs can be compared in terms of these energies. The energy of a DW is defined

FIG. 8. Schematic diagrams of various local structures of DWs. Blue and light green regions indicate the AIAO and AOAI Ir spin ordered domain, respectively. The purple region indicates the DW between two distinct domains. The dashed line indicates the unit cell of \( \text{Nd}_2\text{Ir}_2\text{O}_7 \). (a) Spin structures of the (111) DW consisting of 113O tetrahedra and (b) the (011) DW consisting of 131O tetrahedra. (c) Spin structures of the (100) DW consisting of 212O tetrahedra.
FIG. 9. The $\rho_{xx}$ vs $\rho_{xy}$ plot at 2 K under the $H$ field along the [111] direction, which is close to coercive field $H^{\text{Ir}}_{c}$. The dashed lines are to guide the eyes. Note that both the $x$ axis and $y$ axis are depicted in the log scale.

as the energy cost per area ($A$) and is calculated as $E_{\text{DW}} = 2(E_{2I2O} - E_{\text{AIO}})/A = 16 J_{\text{eff}}/A$ for a (100) DW consisting of Ir-2I2O tetrahedra [see Fig. 8(c)]. Similarly, the [111] and {011} DWs containing only Ir-I3O tetrahedra as depicted in Figs. 8(a) and 8(b) are most stable with $E_{\text{DW}} \sim 13.9 J_{\text{eff}}/A$, and $\sim 17.0 J_{\text{eff}}/A$, respectively.

Among these possible DWs configurations, the [111] DW is the most preferable state due to its lowest $E_{\text{DW}}$ value. Other types of DWs also can be formed, and can be understood as the combination of the above DW configurations. Particularly, within $H$ along [111], the energy cost of (111) plane DW formation should be lower. Therefore, in our experimental geometry, the Nd$_2$Ir$_2$O$_7$ thin film should contain the (111) plane DW with $H$ along the [111] direction.

APPENDIX B: QUADRATIC DEPENDENCE OF THE DOMAIN-WALL ANOMALOUS HALL RESISTIVITY ON LONGITUDINAL RESISTIVITY

The quadratic dependence of the DW anomalous Hall resistivity on the longitudinal resistivity indicates that the intrinsic AHE arises at $H \sim H^{\text{Ir}}_{c}$, where a large DW contribution plays a role. Based on the intrinsic mechanism, $\rho_{xy}$ is known to exhibit a power-law dependence on $\rho_{xx}$, i.e., $\rho_{xy} \sim \rho_{xx}^{a_{xy}}$, with an exponent $a_{xy} \sim 2.0$ [45]. Near the domain switching region, where the DW contribution is largest, the power-law dependence of $\rho_{xy}$ shows $\rho_{xy} \sim \rho_{xx}^{2}$ (see Fig. 9). Ferromagnetic ordering at the DW may induce the intrinsic AHE, which is usually observed in ferromagnets.

APPENDIX C: HALL MEASUREMENT ON OTHER RARE-EARTH PYROCHLORIDE IRIDATE THIN FILMS

We measure the Hall resistivity of a Sm$_2$Ir$_2$O$_7$/YSZ (111) thin film, which has a thickness of $\sim 170$ nm. The current path is taken along the [110] direction, and $H$ is always oriented perpendicular to the current direction, which is along the [111] direction. Figure 10 shows the $\rho_{xy}$ curves at various $T$, including the value at $T_{\text{r}} \sim 120$ K of Sm$_2$Ir$_2$O$_7$. At high temperatures (above $\sim 70$ K), the Hall resistivity has a linear dependence on $H$, which is typical behavior for the ordinary Hall effect. However, at lower temperatures (below $\sim 20$ K), we observe an anomaly that presumably arises due to the Ir moment canting effect. In comparison to the Hall results for the Nd$_2$Ir$_2$O$_7$ thin film, no hysteresis occurs. We speculate that this different Hall behavior is induced by the different DW conductance.

[26] Under the $H$ field, both spin canting and domain switching can induce MR. However, the irreversible processes, such as hysteresis, can be explained in terms of the domain switching.