Axionlike particle assisted strongly interacting massive particle

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We propose a new realization of strongly interacting massive particles (SIMPs) as self-interacting dark matter, where SIMPs couple to the standard model (SM) sector through an axionlike particle. Our model overcomes major obstacles accompanying the original SIMP model, such as a missing mechanism of kinetically equilibrating SIMPs with the SM plasma as well as marginal perturbativity of the chiral Lagrangian density. Remarkably, the parameter region realizing $\sigma_{\mathrm{self}}/m_{\mathrm{DM}} \simeq 0.1\text{–}1\ \mathrm{cm}^2/\mathrm{g}$ is within the reach of future beam dump experiments such as the Search for Hidden Particles experiment.

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I. INTRODUCTION

The presence of dark matter (DM) in the Universe has been firmly established by cosmological observations at scales spanning orders of magnitude, i.e., from the cosmic microwave background (CMB) anisotropies to the stellar velocity dispersion or the rotation curve of dwarf galaxies (see, e.g., Ref. [1]). However, little is currently known about the nature of DM. A weakly interacting massive particle (WIMP) model has been a prominent paradigm, and it can be naturally accommodated in a particle physics model beyond the standard model (SM) that is suggested as a solution to the hierarchy problem (e.g., low-scale supersymmetry [2]). However, despite the extensive efforts thus far, collider experiments, direct detection experiments, and indirect searches have all failed to find any evidence of WIMPs. The WIMP paradigm has been tightly constrained (see, e.g., Ref. [3]).

Furthermore, the conventional WIMP paradigm has also been questioned by observations of the matter distribution of the Universe. The structure formation of WIMPs is concordant with that in the conventional cold DM (CDM) model and thus reproduces the observed structure at large scales. On the other hand, there are reported discrepancies between observed subgalactic scale structures and the CDM predictions, which are collectively called the small-scale crisis (see, e.g., Ref. [4]). One example is the core-cusp problem (see, e.g., Ref. [5]): the CDM model predicts a cuspy inner density profile (inversely proportional to the distance from the center) such as the Navarro-Frenk-White profile [6] for the DM distribution in halos, while observed dwarf galaxies show a cored profile [7–9]. Recent state-of-the-art hydrodynamical simulations in the CDM model, which incorporate dynamical processes of baryons, rediscover the core-cusp problem in a clearer manner: a large part of mass should be expelled from an inner part of halos [10]. These observations may hint at DM properties that conventional WIMP models do not offer.

Self-interacting DM (SIDM) is one of the most intriguing possibilities as a solution to the core-cusp problem [11]. With a self-scattering cross section per DM mass of $\sigma_{\mathrm{self}}/m_{\mathrm{DM}} \simeq 0.1\text{–}1\ \mathrm{cm}^2/\mathrm{g}$, DM particles in an inner part of halos are thermalized within a dynamical time scale of halos, which leads to a lower mass density and a core DM profile [12–15]. SIDM also alleviates the unexpected diversity problem [16,17] found in the aforementioned hydrodynamical simulations [10]: the simulations predict similar rotation curves for similar-size dwarf galaxies, while the observed rotation curves show diversity. The SIDM profile is more amenable to a baryon profile that possesses diversity even among similar-size halos, when compared to the CDM profile [18,19].

On the other hand, the DM self-scattering also renders the shape of DM halos more spherical. The observed ellipticity of the DM halos in galaxy clusters constrains the self-scattering cross section: $\sigma_{\mathrm{self}}/m_{\mathrm{DM}} \lesssim 0.1\ \mathrm{cm}^2/\mathrm{g}$ [20]. Bullet clusters imply that the colliding DM halos should pass by each other and thus place an upper bound: $\sigma_{\mathrm{self}}/m_{\mathrm{DM}} \lesssim 0.7\ \mathrm{cm}^2/\mathrm{g}$ [21,22]. To reconcile these constraints from galaxy clusters and the required self-scattering cross section from the small-scale crisis at the galaxy scale or below, it has been claimed that the cross section should be velocity dependent and it diminishes with an increasing relative velocity (see, e.g., Ref. [23]). These constraints, however, are vulnerable to modeling and/or statistical uncertainties. In fact, $\sigma_{\mathrm{self}}/m_{\mathrm{DM}}$ inferred from a bullet cluster (Abell 3827) may be changed by orders of magnitude depending on analyses [24,25]. Thus in this paper we take a relatively wider range of $\sigma_{\mathrm{self}}/m_{\mathrm{DM}} \simeq 0.1\text{–}1\ \mathrm{cm}^2/\mathrm{g}$.

Particle physics aspects of SIDM are yet to be examined, specifically, how such DM can be accommodated in a concrete particle physics model as well as what a viable thermal history at an early epoch of the Universe is. References [26,27] proposed a model of a strongly

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interacting massive particle (SIMP), where pions (π's) in a hidden confinement sector are identified as DM. The exact unbroken flavor symmetry may ensure the longevity of the pions. However, there are shortcomings in the model: necessity of a kinetic equilibration mechanism and marginal perturbativity. First, the $3 \to 2$ process induced by the Wess-Zumino-Witten (WZW) term [28,29] reduces the number density of pions to the observed DM mass density. A major assumption in the Boltzmann equation of the aforementioned literature [26,27] is that the DM temperature scales as that of the SM plasma. With the $3 \to 2$ process and the self-scattering, however, the DM temperature scales only inversely logarithmically with that of the SM plasma [30]. To this end, we need to maintain kinetic equilibrium between the SIMP and the SM plasma. This is a missing piece in the original literature, while possible kinetic equilibration mechanisms, such as kinetic mixing portal [31,32] and a Higgs portal [33], were later suggested. Second, the pion mass per pion decay constant is around the perturbativity bound of the naïve dimensional analysis ($2\pi/\sqrt{N_c}$) [34,35] in the parameter region where the observed DM abundance and self-interaction are obtained. The higher-order terms of the chiral Lagrangian density may cause an order-one change in the result [36].

We address these issues by considering an axionlike particle (ALP, which we denote as ϕ) portal in the hidden confinement sector model.[1] The relic density is dominantly determined by a semiannihilation [41] ($\pi\pi \to \pi\phi$) rather than the $3 \to 2$ process. ALPs are well thermalized with the SM plasma and transfer kinetic energies between the pions and the SM plasma through the semiannihilation. We find that this mechanism works when the ALP mass is degenerate with the pion mass and the ALP decay constant is just above the electroweak scale.

This paper is organized as follows. In the next section, we introduce a hidden sector that incorporates an ALP as well as SIMPs, where we take into account the CP-violating terms. In Sec. III, we give formulas of cross sections relevant to the DM phenomenology. Furthermore, we address how primary concerns in the original SIMP framework can be solved in our setup. Current constraints and future detectability of our model are presented in Sec. IV. We conclude in the final section. In Appendix, we discuss how our model can be generalized to gauge groups other than $SU(N_c)$. We also provide formulas of group factors that appear in the formulas of the cross sections.

II. HIDDEN SECTOR WITH AN ALP

In this paper, we consider the following Lagrangian density describing the hidden sector,

[1] These points are also examined in different setups [37–40].

$$\begin{aligned}\mathcal{L}_{\text{hid}} = {}& \frac{1}{2}(\partial_\mu \phi)^2 - V_{\text{UV}}(\phi) + N_L^\dagger i\bar{\sigma}^\mu D_\mu N_L + \bar{N}_L^\dagger i\bar{\sigma}^\mu D_\mu \bar{N}_L \\ & - m_N(\bar{N}_L N_L + \text{H.c.}) - \frac{1}{4}H^{i\mu\nu}H^i{}_{\mu\nu} \\ & + \frac{g_H^2}{32\pi^2}\left(\frac{\phi}{f} + \theta_H\right)H^i{}_{\mu\nu}\tilde{H}^{i\mu\nu}, \end{aligned} \tag{1}$$

where N_L and $\bar{N}_L$ are N_f-flavored ($N_f \geq 3$) vectorlike fermion pairs, which respectively transform as the fundamental and antifundamental representations of a $SU(N_c)$ ($N_c \geq 2$) gauge group, and $H^i_{\mu\nu}$ ($\tilde{H}^i_{\mu\nu}$) is the (dual) field strength of a hidden gauge field. The decay constant of an ALP (ϕ) is denoted by f and $V_{\text{UV}}(\phi)$ is a contribution to the potential of ϕ from an underlying model. It is convenient to make a chiral rotation of N_f vectorlike fermions to eliminate the theta angle in front of $H^i{}_{\mu\nu}\tilde{H}^{i\mu\nu}$. After such a chiral rotation, we see that the mass matrix becomes $m_N \to m_\theta = m_N e^{i\theta_H/N_f}$. We assume that the gauge interaction confines the vectorlike fermions below some energy scale (μ), which is sufficiently larger than the fermion mass m_N (we define θ_H so that m_N is real). The fermions form a condensate so that the flavor symmetry breaks as $SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V$. Let us again remark that the unbroken flavor symmetry, $SU(N_f)_V$, is essential for longevity of pions. We parametrize the fermion bilinear as $N_{Li}\bar{N}_{Lj} = \mu^3 \tilde{U}_{ij}$ with a $U(N_f)$-valued field of

$$\tilde{U} = U \exp\left[2i\frac{\eta'}{f_{\eta'}}\sqrt{\frac{2}{N_f}}\right], \qquad U = \exp\left[\frac{2i\pi^a T^a}{f_\pi}\right], \tag{2}$$

where we introduce Nambu-Goldstone bosons (π's and η') and their decay constants (f_π and $f_{\eta'}$). Here T^a [$a = 1, \ldots, N_\pi (= N_f^2 - 1)$] are the generators of $SU(N_f)_A$ normalized as $\text{Tr}(T^a T^b) = 2\delta_{ab}$.

It is expected that the chiral anomaly provides a potential for the linear combination of $\phi/f - 2\sqrt{2N_f}\eta'/f_{\eta'}$ and the resultant η' mass is higher than pions [42,43]. After integrating out η', we obtain the following chiral Lagrangian density,

$$\mathcal{L}_{\text{hid}} \supset \frac{f_\pi^2}{16}\text{Tr}(\partial_\mu U \partial^\mu U^\dagger) + \mu^3 \text{Tr}(m_\theta \tilde{U} + \text{H.c.}) + \mathcal{L}_{\text{WZW}}, \tag{3}$$

with $\tilde{U} = U e^{i\phi/(N_f f)}$. The last term is called the WZW term [28,29], which introduces the $3 \leftrightarrow 2$ interaction of pions [27].

There is no reason why the minimum of the ALP UV potential should be aligned with that of the potential originating from $H^i{}_{\mu\nu}\tilde{H}^{i\mu\nu}$ and the CP symmetry is respected. Once we assume that ϕ dominantly obtains the mass from $V_{\rm UV}(\phi)$, the theory violates CP symmetry. We define θ_H so that $V_{\rm UV}(\phi)$ takes the minimum at $\phi = 0$, and then the order parameter for CP violation is given as $\mathrm{Im}(m_\theta) \propto \sin(\theta_H/N_f)$. We remark that periodicity of $\theta_H \to \theta_H + 2\pi n$ (n: integer) is maintained in the chiral Lagrangian density since $\exp(2\pi i n/N_f) \in \mathrm{SU}(N_f)$ and thus can be eliminated by a chiral transformation of π's.

We expand the matrix of $\tilde{U}$ to obtain the following Lagrangian density of the pions and the ALP,

$$\mathcal{L}_{\rm hid} = \mathcal{L}_0 + \mathcal{L}_{\rm CP} + \mathcal{L}_{\rm CPV} + \mathcal{L}_{\rm WZW}, \tag{4}$$

where

$$\begin{aligned}
\mathcal{L}_0 &= \frac{1}{2}(\partial_\mu \pi^a)^2 + \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m_\pi^2 (\pi^a)^2 - \frac{1}{2}m_\phi^2 \phi^2, \\
\mathcal{L}_{\rm CP} &= \frac{m_\pi^2}{4N_f^2 f^2}(\pi^a)^2\phi^2 - \frac{1}{6f_\pi^2} r_{abcd}(\partial_\mu \pi^a)(\partial^\mu \pi^b)\pi^c\pi^d \\
&\quad + \frac{m_\pi^2}{6N_f f_\pi f} d_{abc}\pi^a\pi^b\pi^c\phi + \frac{m_\pi^2}{12 f_\pi^2} c_{abcd}\pi^a\pi^b\pi^c\pi^d, \\
\mathcal{L}_{\rm CPV} &= \tan(\theta_H/N_f)\left[\frac{m_\pi^2}{2N_f f}\phi(\pi^a)^2 \right.\\
&\quad \left. + \frac{m_\pi^2}{6f_\pi} d_{abc}\pi^a\pi^b\pi^c - \frac{m_\pi^2}{30 f_\pi^3}\pi^a\pi^b\pi^c\pi^d\pi^e c_{abcde}\right], \\
\mathcal{L}_{\rm WZW} &= \frac{2N_c}{15\pi^2 f_\pi^5}\epsilon^{\mu\nu\rho\sigma} c_{[abcde]}(\pi^a\partial_\mu\pi^b\partial_\nu\pi^c\partial_\rho\pi^d\partial_\sigma\pi^e).
\end{aligned} \tag{5}$$

Here, we define $m_\pi^2 f_\pi^2 = 16 m_N \mu^3 \cos(\theta_H/N_f)$. The ALP mass ($m_\phi$) also receives a contribution from the UV potential as mentioned above: $m_\phi^2 \geq m_\pi^2 f_\pi^2/(8N_f f^2)$. In the above expression, we have kept only relevant terms. Group factors such as d^2 are defined as given in Eqs. (A1)–(A13) in Appendix. A (square) parenthesis in a sub/superscript represents the total (anti)symmetrization of the enclosed indices.

In addition, we assume that ϕ couples to the SM sector via the following Lagrangian density,

$$\mathcal{L}_{\phi\gamma\gamma} = C_{\phi\gamma\gamma}\frac{\alpha}{4\pi}\frac{\phi}{f}F_{\mu\nu}\tilde{F}^{\mu\nu}, \tag{6}$$

where α is the fine structure constant and $F^{\mu\nu}$ ($\tilde{F}^{\mu\nu}$) is the (dual) field strength of the photon. $C_{\phi\gamma\gamma}$ is a constant typically of order unity depending on the underlying model.

III. DARK MATTER PHENOMENOLOGY

First, let us compute the flavor-averaged cross sections of the following processes relevant to the pion freeze-out in the early Universe and their self-interactions in DM halos at a later era: the semiannihilation ($\pi\pi \to \pi\phi$), the $3 \to 2$ process ($\pi\pi\pi \to \pi\pi$), and the self-scattering ($\pi\pi \to \pi\pi$). We assume that the initial (left-hand side) pions are nonrelativistic. Next, we describe the roles that the semiannihilation plays during the pion freeze-out, especially stressing that the semiannihilation contributes to the kinetic equilibration between the pions and the SM plasma when the masses of the pions and the ALP are degenerate: $m_\pi \simeq m_\phi$. We also show to what extent the semiannihilation helps us to mitigate the perturbativity issue.

The cross section of the semiannihilation ($\pi\pi \to \pi\phi$) is given by

$$\begin{aligned}
\langle \sigma_{\rm semi} v_{\rm rel}\rangle &\simeq \frac{1}{64\pi}\frac{m_\pi^2}{f_\pi^2 f^2}\frac{d^2}{N_f^2 N_\pi^2}\mathcal{I}(m_\phi, m_\pi, T) \\
&\quad \times \left[1 + \frac{m_\pi^2 + m_\phi^2/9}{m_\pi^2 - m_\phi^2/3}\tan^2(\theta_H/N_f)\right]^2,
\end{aligned} \tag{7}$$

where $d^2 = \sum_{abc} d_{abc}^2$ [see also Eqs. (A1)–(A13) in Appendix]. The brackets denote the thermal average with temperature of T. If $m_\phi \ll m_\pi$, the phase space factor [$\mathcal{I}(m_\phi, m_\pi, T)$] is given by

$$\mathcal{I} = \frac{3}{4}\sqrt{\left(1 - \frac{m_\phi^2}{9m_\pi^2}\right)\left(1 - \frac{m_\phi^2}{m_\pi^2}\right)}\left[\frac{K_1(m_\pi/T)}{K_2(m_\pi/T)}\right]^2, \tag{8}$$

where $K_n(x)$ is the nth-order modified Bessel function of the second kind. On the other hand, if the masses are degenerate ($m_\phi = m_\pi$), we find

$$\mathcal{I} = \frac{2T}{m_\pi}\frac{K_1(2m_\pi/T)}{K_2^2(m_\pi/T)}, \tag{9}$$

which can be approximated as $\mathcal{I} \approx 2/\sqrt{\pi x} = v_{\rm rel}/2$ for $x = m_\pi/T \gg 1$.

The cross sections of the $3 \to 2$ process ($\pi\pi\pi \to \pi\pi$) and the self-scattering ($\pi\pi \to \pi\pi$) are found in the original model [27], but they need to be extended to incorporate a nonzero CP phase. We obtain

$$\begin{aligned}
\langle \sigma_{3\to 2} v_{\rm rel}^2\rangle &= \frac{5\sqrt{5}}{2\pi^5}\frac{N_c^2 m_\pi^5 t^2}{f_\pi^{10} N_\pi^3 x^2} \\
&\quad + \frac{\sqrt{5}}{2304\pi}\tan^2(\theta_H/N_f)\frac{m_\pi}{N_\pi^3 f_\pi^6}\{CD\}^2,
\end{aligned} \tag{10}$$

$$\begin{aligned}
\sigma_{\rm self} &= \frac{1}{32\pi}\frac{m_\pi^2}{N_\pi^2 f_\pi^4}\left[\{C+R\}^2 - \tan^2(\theta_H/N_f)\right. \\
&\quad \left. \times \{C+R\}D^2 + \tan^4(\theta_H/N_f)\frac{D^4}{4}\right],
\end{aligned} \tag{11}$$

where again group factors such as t^2 are defined in Eqs. (A1)–(A13) of Appendix.

Let us recall that f should be substantially larger than f_π so that the Lagrangian density in Eq. (1) is valid. It follows that the other processes such as the annihilation of a pion pair into the ALPs ($\pi\pi \to \phi\phi$) and the scattering of the pion with the ALP ($\pi\phi \to \pi\phi$) are not relevant in the course of the pion freeze-out. For example, $\pi\pi \to \phi\phi$ contributes to the chemical equilibration between the pions and the ALPs, but decouples earlier than $\pi\pi \to \pi\phi$, since the cross section is suppressed by $(f_\pi/f)^2$ when compared to $\langle \sigma_{\rm semi} v_{\rm rel} \rangle$. For the same reason, we find that $\pi\phi \to \pi\phi$ is not efficient enough to keep the pions in kinetic equilibrium with the SM plasma during the pion freeze-out.

Whether the semiannihilation or the $3 \to 2$ process dominates the chemical equilibration depends on the masses and the decay constants of the pions and the ALP. Either process, in general, leads to a conversion of the DM mass energy into the kinetic energy. Unless DM particles can efficiently deposit the injected kinetic energy into the SM plasma, DM particles are heated up in the course of the pion freeze-out. In such a case, kinetic equilibrium is hardly maintained between the DM particles and the SM plasma, and the evolution of the DM temperature hence becomes far from obvious. Although the temperature evolution out of kinetic equilibrium is worth investigating, we leave this for a future study [44], and in the rest of this paper we focus on the case where the masses are degenerate between the pions and the ALP, i.e., $m_\pi \simeq m_\phi$ and the semiannihilation dominates the $3 \to 2$ process. Assuming the degenerate masses, we omit the conversion of a mass deficit into the kinetic energy from the semiannihilation. As a consequence, the semiannihilation now contributes to the kinetic equilibration between the DM particles and the SM plasma as well as the chemical one. Thus, the DM freeze-out in our scenario proceeds in the same manner as the semiannihilating DM model (see, e.g., Ref. [41]). Furthermore, as we see closely in the next section, the degenerate masses help our DM pions to evade constraints from indirect searches for the semiannihilation at a later epoch of the Universe.

The domination of the semiannihilation also helps us to alleviate the issue regarding perturbativity in the original model. To see this, we take $N_f = 4$, $\theta_H = 0$, $\alpha_\pi = m_\pi/f_\pi = 2$, and $f = 200$ GeV as a benchmark point (denoted by $\star$ in Fig. 1). At this benchmark point, we obtain

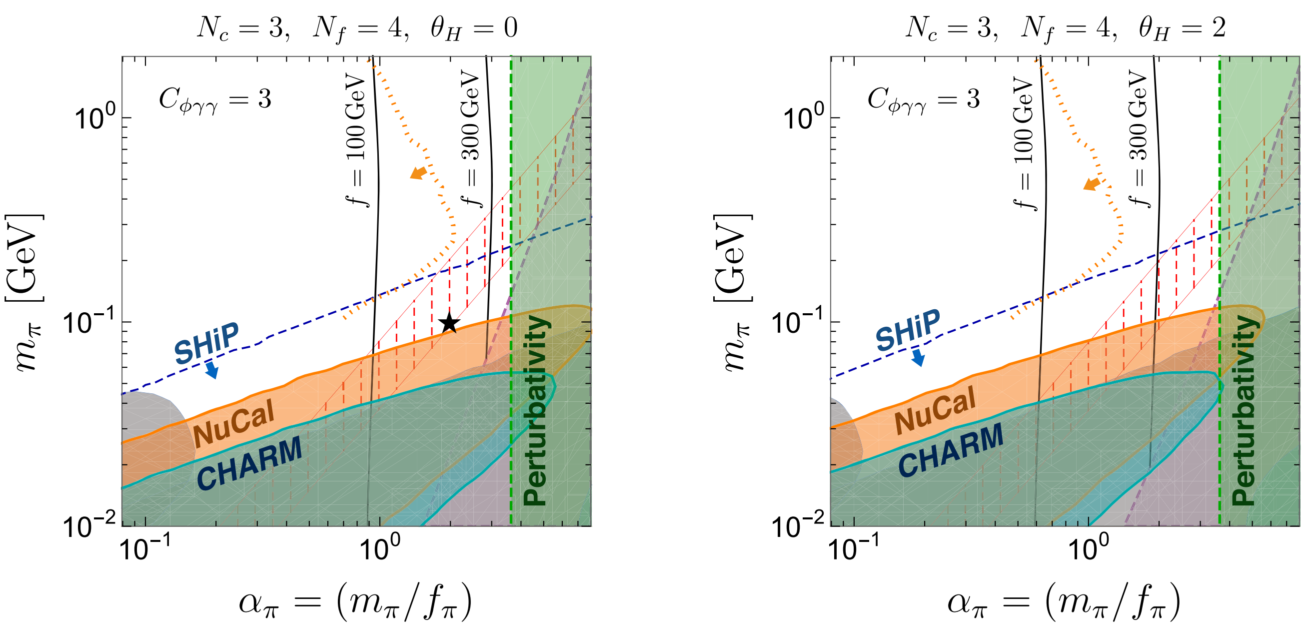

FIG. 1. Shown are m_π–α_π planes of the hidden sector. We assume that $N_c = 3$, $N_f = 4$, and $C_{\phi\gamma\gamma} = 3$. θ_H is taken to be 0 (left panel) and 2 (right panel). Black lines give the observed DM abundance ($\Omega_{\rm DM} h^2 = 0.12$) for $f = 100$ and 300 GeV from the left to the right in each panel. In the red hatched band, the pion self-scattering achieves the SIDM cross section, i.e., $0.1\ {\rm cm}^2/{\rm g} \le \sigma_{\rm self}/m_\pi \le 1\ {\rm cm}^2/{\rm g}$. In the magenta shaded region, the $3 \to 2$ process dominates the semiannihilation and thus determines the pion freeze-out. In the green region, the pion mass exceeds a naïve cutoff scale of the chiral Lagrangian density [see Eq. (3)] [34,35]: $m_\pi \ge 2\pi f_\pi/\sqrt{N_c}$. Constraints on the ALP from NuCal (orange), CHARM (dark cyan), SLAC E-137 & E-141 (gray), and SN1987A (light cyan at the bottom right corner) are also shown. We convert the constraints to those on the m_π–α_π plane by imposing $m_\phi = m_\pi$ so as not to heat up the DM pions through the semiannihilation in the course of the chemical freeze-out and by regarding f as a function of m_π and f_π determined by the observed DM abundance. The orange dotted line is the projected sensitivity of the Belle II experiment for an ALP search [45]. The projected SHiP experiment will examine the region below the blue dashed lines, which covers a large part of the parameter space where the observed DM abundance and the SIDM cross section are simultaneously achieved.

$$\langle\sigma_{\rm semi} v_{\rm rel}\rangle|_{T=T_{\rm fo}} \simeq \frac{6\times 10^{-9}}{\rm GeV^2}\left(\frac{\alpha_\pi}{2}\right)^2\left(\frac{200\ \rm GeV}{f}\right)^2 \times\sqrt{\frac{19}{x_{\rm fo}}},$$

$$\langle\sigma_{3\to 2} v_{\rm rel}^2\rangle|_{T=T_{\rm fo}} n_{\rm fo} \simeq \frac{4\times 10^{-11}}{\rm GeV^2}\left(\frac{\alpha_\pi}{2}\right)^{10}\left(\frac{100\ \rm MeV}{m_\pi}\right)^6 \times\left(\frac{N_c}{3}\right)^2\left(\frac{19}{x_{\rm fo}}\right)^2,$$

$$\frac{\sigma_{\rm self}}{m_\pi} \simeq 0.9\ {\rm cm^2/g}\left(\frac{\alpha_\pi}{2}\right)^4\left(\frac{100\ \rm MeV}{m_\pi}\right)^3,$$

where $n_{\rm fo}$ and $T_{\rm fo}$ respectively denote the freeze-out number density, which is determined by the observed DM density, and temperature at the pion freeze-out. For $\alpha_\pi = 2$ (below the perturbativity bound) and $\sigma_{\rm self}/m_\pi = 1\ {\rm cm^2/g}$ (the SIDM cross section), the cross section of the $3\to 2$ process ($\pi\pi\pi\to\pi\pi$) is too small to provide the observed DM mass density (recall the canonical WIMP cross section: $\langle\sigma v_{\rm rel}\rangle_{\rm can} \simeq 3\times 10^{-9}/{\rm GeV^2}$). The semiannihilation cross section, on the other hand, takes the appropriate value to result in the observed DM mass density, provided that the ALP decay constant is around the electroweak scale. Figure 1 shows the parameter regions where we obtain the observed DM abundance ($\Omega_{\rm DM}h^2 \simeq 0.12$, black line), the self-scattering cross section for SIDM ($0.1\ {\rm cm^2/g} \le \sigma_{\rm self}/m_\pi \le 1\ {\rm cm^2/g}$, red hatched), and the semiannihilation cross section going below that of the $3\to 2$ process (magenta shaded). As long as the semiannihilation dominates the $3\to 2$ process, the observed DM abundance can be realized with the appropriate value of f. The figure shows that our model reconciles marginal perturbativity with the SIDM cross section. Note that in our model setup, the self-scattering cross section is velocity independent. This is a generic feature in SIMP models where hidden pions are regarded as SIDM [26,27]. If the constraints on the self-scattering cross section from galaxy clusters [20–22] are taken to be robust, pions can still account for DM but cannot alleviate the small-scale crisis that requires a larger cross section than constrained.

Our argument on the pion freeze-out so far relies on a few implicit assumptions, which we clarify now. First, the ALPs are assumed to be thermalized with the SM plasma at least until the freeze-out of DM. The Primakoff process and the decay and inverse decay through the interaction given in Eq. (6) are responsible for thermalization of the ALPs. In particular, the decay and inverse decay are efficient in the course of the pion freeze-out. When the ALPs are relativistic, the rate of the decay and inverse decay is approximately given by

$$\langle\Gamma_{\rm dec}\rangle \simeq C_{\phi\gamma\gamma}^2\frac{\alpha^2}{768\pi\zeta(3)}\frac{m_\phi^4}{f^2 T}, \tag{12}$$

where $\zeta(x)$ is the Riemann zeta function. The recoupling temperature, below which the decay and inverse decay are efficient, is estimated as

$$T_{\rm rec} \simeq 2\ {\rm GeV}\, C_{\phi\gamma\gamma}^{2/3}\left(\frac{m_\phi}{100\ \rm MeV}\right)^{4/3}\left(\frac{200\ \rm GeV}{f}\right)^{2/3}. \tag{13}$$

We find that thermalization of the ALPs is guaranteed in the parameter region allowed by the existing constraints, which are discussed in the next section and shown in Fig. 1.

Second, without a further extension of the hidden sector, the pions need to be produced efficiently from the SM plasma after the Universe is reheated. Even if the reheating temperature is very low, for example, $T_{\rm rh} \lesssim (2\pi/\sqrt{N_c})f_\pi$, pions can be produced through $\phi\phi\to\pi\pi$ first. Its cross section is given by

$$\langle\sigma_{\phi\phi\to\pi\pi} v_{\rm rel}\rangle \simeq \frac{1}{128\pi}\frac{m_\pi^2}{N_f^4 f^4}[1+\tan^2(\theta_H/N_f)]^2 \times\mathcal{I}(m_\phi, m_\pi, T), \tag{14}$$

when the ALPs are nonrelativistic and $m_\pi \simeq m_\phi$ [see Eq. (9) for $\mathcal{I}$]. The number of the $\phi\phi\to\pi\pi$ reactions per Hubble time can be smaller than unity,

$$\langle\sigma_{\phi\phi\to\pi\pi} v_{\rm rel}\rangle n_\phi/H \simeq 0.03\left(\frac{m_\pi}{100\ \rm MeV}\right)^3\left(\frac{200\ \rm GeV}{f}\right)^4 \times\left(\frac{T}{m_\pi}\right)^{3/2} e^{-m_\pi/T}, \tag{15}$$

where we take $N_f = 4$ and $\theta_H = 0$ again. However, we remark that the efficient semiannihilation multiplies the produced number of pions by the exponential of

$$\langle\sigma_{\rm semi} v_{\rm rel}\rangle N_\pi n_\phi/H \simeq 2\times 10^8\left(\frac{\alpha_\pi}{2}\right)^2\left(\frac{m_\pi}{100\ \rm MeV}\right) \times\left(\frac{200\ \rm GeV}{f}\right)^2\left(\frac{T}{m_\pi}\right)^{3/2} e^{-m_\pi/T}. \tag{16}$$

The above two observations indicate that in the viable parameter region shown in Fig. 1, the pions can be efficiently sourced as long as the reheating temperature is larger than $T_{\rm fo}$.

IV. CONSTRAINTS AND FUTURE DETECTABILITY

The coupling of Eq. (6), which plays an important role in the kinetic equilibration between the pions and the SM plasma, is subject to various constraints on an ALP. Among them, the most stringent constraints to our model come from beam dump experiments, which are discussed shortly below. We depict these constraints in the m_π–α_π plane of Fig. 1, by requiring $m_\phi = m_\pi$ to suppress the conversion of the mass deficit into the DM kinetic energy and determining f to reproduce the observed DM abundance. In the figure, we adopt $C_{\phi\gamma\gamma} = 3$ as a representative value.

Beam dump experiments constrain an ALP by exploiting the production of ALPs through the Primakoff process of a virtual photon. Depending on the beam energy and the baseline distance, those experiments are sensitive to a particular range of the ALP lifetime and mass. Proton beam dump experiments, such as the CERN-Hamburg-Amsterdam-Rome-Moscow Collaboration (CHARM) [46] and NuCal [47], as well as electron beam dump experiments, such as Stanford Linear Accelerator Center (SLAC) E-137 & E-141 [48,49], constrain the coupling of a MeV-scale ALP to photons. Search for Hidden Particles (SHiP) is a projected proton beam dump experiment [50]. The constraints of these experiments are plotted in Fig. 1 based on Ref. [51]. From Fig. 1, one can observe that our model evades the existing constraints when $m_\pi = m_\phi \gtrsim 40$ MeV. In particular, when the pion mass is around 100 MeV and α_π is a few, our pion DM has a sizable self-scattering cross section that is compatible with SIDM hinted by the small-scale crisis. The lower mass region ($m_\pi = m_\phi < 100$ MeV) is excluded mainly by NuCal, which has a relatively short baseline. The lower bound on the mass tends to be relaxed as $C_{\phi\gamma\gamma}$ increases, since a produced ALP tends to decay before it reaches a decay volume. Our model will be better probed by future beam dump experiments with increased sensitivity to a shorter-lived ALP. Actually, as shown in Fig. 1, the SHiP experiment will be able to probe a substantial part of the parameter region where SIDM is realized in our model. Furthermore, future B factories such as the Belle II experiment [52] will cover a higher mass region ($m_\pi = m_\phi > 100$ MeV) [45] and will be complementary to beam dump experiments, as shown in Fig. 1.

Let us comment on an underlying model-dependent implication of an electroweak-scale ALP decay constant. We can realize the ALP considered in this paper by introducing heavy vectorlike fermions and scalar fields charged under the corresponding anomalous global symmetry as in the Kim-Shifman-Vainshtein-Zakharov axion model [53,54]. The heavy fermions carry a hypercharge and transform as some representation of $\mathrm{SU}(N_c)$ and the confining gauge group that are responsible for $V_{\mathrm{UV}}(\phi)$ in Eq. (1). In this case, the coupling of Eq. (6) originates from an ALP coupling to the hyperchage gauge boson, and the Large Electron-Positron Collider disfavors an ALP decay constant of $f/C_{\phi\gamma\gamma} \lesssim 3$ GeV (from the $Z \to \gamma\phi$, $\phi \to 2\gamma$ decay) for an ALP with mass around 100 MeV [55]. Note that the constraint may be improved by orders of magnitude in future lepton colliders such as the International Linear Collider [56], the Circular Electron Positron Collider, and the Future Circular Collider [57]. In the case where the heavy fermions transform as some representation of the weak SU(2) instead of the hypercharge, the induced flavor-changing neutral current process ($K^\pm \to \pi^\pm\phi$, $\phi \to 2\gamma$) constrains the ALP decay constant: $f/C_{\phi\gamma\gamma} \gtrsim 170$ GeV for $m_\pi \sim 100$ MeV [45]. Furthermore, the newly introduced heavy fermions may also be subject to collider constraints. The heavy fermion mass is related with the ALP decay constant as $m_{\mathrm{fermion}} = Yf\sum_r 2T(r)$, where Y is a Yukawa coupling between the heavy vectorlike fermions and the scalar fields. We also define $T(r)$ by $\mathrm{Tr}(T_r^i T_r^j) = T(r)\delta_{ij}$ with the generator (T_r) of the representation (r), where we normalize the structure constant of $\mathrm{SU}(N_c)$ so that $T(r) = 1/2$ for the fundamental representation. It follows that the heavy fermions may evade constraints from collider experiments, depending on the choice of the representation, which may lead to m_{fermion} as heavy as $\sim$1 TeV, and/or the hypercharge. Therefore, we just summarize the current status of searches for long-lived charged particles produced from the Drell-Yan process in the Large Hadron Collider: $m_{\mathrm{fermion}} > 650$ GeV with a charge unity in units of the positron charge [58] and $m_{\mathrm{fermion}} > 310(140)$ GeV with a charge $2/3$ $(1/3)$ [59].

Besides the collider experiments, an ALP with mass around 100 MeV is constrained also by supernovae (SNe) since the energy loss rate of SNe is enlarged by emission of ALPs [60]. This constraint is relevant when the ALP mass is sufficiently light such that ALPs are thermally produced in a SN core where the temperature is about 50 MeV and the coupling is in the range where ALPs are copiously produced but are not trapped in the core. We find that in our model, the constraint from SN1987A becomes important only when $C_{\phi\gamma\gamma}$ is as small as 0.01.

Finally, we discuss aspects in indirect DM searches. In our model setup, where $m_\phi \simeq m_\pi$ is required to avoid a possible heating of the DM pions through the semiannihilation, a pair of pions semiannihilates into an ALP that subsequently decays into two photons with energy of $m_\pi/2$. Such a late-time semiannihilation potentially affects, for example, galactic and extragalactic gamma rays (see, e.g., Refs. [2,61]) as well as the CMB anisotropies [62,63]. Here note that the semiannihilation cross section is proportional to the relative velocity [see Eqs. (7) and (9)] when $\Delta m = m_\pi - m_\phi$ is exactly 0. Therefore, its effect at a later epoch of the Universe, when $v_{\mathrm{rel}} = v_{\mathrm{rel,obs}} < v_{\mathrm{rel,fo}}$, with $v_{\mathrm{rel,fo}} \simeq 2 \times 10^5\ \mathrm{km/s}\sqrt{19/x_{\mathrm{fo}}}$ being the relative velocity at the pion freeze-out, is suppressed.

Let us take a closer look at the case where $m_\pi \gg |\Delta m| \neq 0$. For $\Delta m > 0$, by comparing Eqs. (8) and (9), we find that the semiannihilation cross section scales as $\langle\sigma_{\mathrm{semi}} v_{\mathrm{rel}}\rangle = \langle\sigma_{\mathrm{semi}} v_{\mathrm{rel}}\rangle_{\mathrm{fo}} v_{\mathrm{rel}}/v_{\mathrm{rel,fo}}$ as long as $v_{\mathrm{rel}} > v_{\mathrm{rel,sat}} = 2\sqrt{|\Delta m|/m_\pi}$. An observational upper bound on the semiannihilation cross section denoted by $\langle\sigma_{\mathrm{semi}} v_{\mathrm{rel}}\rangle_{\mathrm{obs}}$ restricts our model to satisfy the following conditions: $v_{\mathrm{rel,sat}}$ and $v_{\mathrm{rel,obs}}$ should be smaller than $v_{\mathrm{rel,fo}}(\langle\sigma_{\mathrm{semi}} v_{\mathrm{rel}}\rangle_{\mathrm{obs}}/\langle\sigma_{\mathrm{semi}} v_{\mathrm{rel}}\rangle_{\mathrm{fo}})$. The first condition

($v_{\mathrm{rel,sat}}/v_{\mathrm{rel,fo}} < \langle\sigma_{\mathrm{semi}} v_{\mathrm{rel}}\rangle_{\mathrm{obs}}/\langle\sigma_{\mathrm{semi}} v_{\mathrm{rel}}\rangle_{\mathrm{fo}}$) implies the required mass difference,

$$\frac{\Delta m}{m_\pi} < 0.07\left(\frac{\langle\sigma_{\mathrm{semi}} v_{\mathrm{rel}}\rangle_{\mathrm{obs}}}{\langle\sigma_{\mathrm{semi}} v_{\mathrm{rel}}\rangle_{\mathrm{fo}}}\right)^2\left(\frac{19}{x_{\mathrm{fo}}}\right). \quad (17)$$

The CMB anisotropies constrain the semiannihilation cross section around and after the last scattering, where $v_{\mathrm{rel,obs}} \lesssim 2\times10^{-4} v_{\mathrm{rel,fo}}\sqrt{100\ \mathrm{MeV}/m_\pi}\sqrt{x_{\mathrm{fo}}/19}$, as $\langle\sigma_{\mathrm{semi}} v_{\mathrm{rel}}\rangle_{\mathrm{obs}}/\langle\sigma_{\mathrm{semi}} v_{\mathrm{rel}}\rangle_{\mathrm{fo}} \lesssim 0.01$–$0.1$ in the mass range of $m_\pi \simeq 0.1$–1 GeV [64–66]. The second condition ($v_{\mathrm{rel,obs}}/v_{\mathrm{rel,fo}} < \langle\sigma_{\mathrm{semi}} v_{\mathrm{rel}}\rangle_{\mathrm{obs}}/\langle\sigma_{\mathrm{semi}} v_{\mathrm{rel}}\rangle_{\mathrm{fo}}$) is trivially satisfied in this case, while the mass difference should be at maximum at a $10^{-(3\text{–}5)}$ level from the first condition. A tighter bound is placed by gamma-ray searches from the Galactic center (GC) in the Energetic Gamma Ray Experiment Telescope (EGRET) [67] and the Fermi Large Area Telescope (Fermi-LAT) [68] for the DM mass larger than 100 (EGRET) and 200 MeV (Fermi-LAT): $\langle\sigma_{\mathrm{semi}} v_{\mathrm{rel}}\rangle_{\mathrm{obs}}/\langle\sigma_{\mathrm{semi}} v_{\mathrm{rel}}\rangle_{\mathrm{fo}} \lesssim 10^{-(2\text{–}3)}$ for the isothermal profile. Note that SIMP possesses a sizable self-scattering cross section and reduces the DM mass density in an inner part of halos [12–15]. The first condition constrains the mass difference at a $10^{-(5\text{–}7)}$ level at maximum. It is unclear whether the second condition is satisfied: $v_{\mathrm{rel,obs}} \lesssim 200$–$2000\ \mathrm{km/s}\sqrt{19/x_{\mathrm{fo}}}$. This is because the DM velocity dispersion in the GC is poorly constrained (especially inside 10 kpc) [69]. Thus we do not show indirect detection constraints in Fig. 1. Future cosmic gamma-ray searches with increased sensitivity to MeV-GeV photons such as e-ASTROGAM [70] may cover the lower mass region ($m_\pi < 100$ MeV) where the observed DM abundance and the SIDM cross section are realized.

For $\Delta m < 0$, the semiannihilation is forbidden [71] and the thermally averaged cross section is further suppressed effectively by a Boltzmann factor of $\exp(\Delta m/T) = \exp[-(4/\pi)v_{\mathrm{rel,sat}}^2/v_{\mathrm{rel}}^2]$. Let us assume that $|\Delta m| \ll T_{\mathrm{fo}} = m_\pi/x_{\mathrm{fo}}$, i.e., $v_{\mathrm{rel,sat}} \ll 10^5\ \mathrm{km/s}\sqrt{19/x_{\mathrm{fo}}}$, to ignore the suppression during the pion freeze-out and keep the discussion in the previous section intact. In this case, the condition of

$$\max[v_{\mathrm{rel,sat}}, v_{\mathrm{rel,obs}}]\exp\left[-\frac{4}{\pi}\frac{v_{\mathrm{rel,sat}}^2}{v_{\mathrm{rel,obs}}^2}\right] < v_{\mathrm{rel,fo}}\left(\frac{\langle\sigma_{\mathrm{semi}} v_{\mathrm{rel}}\rangle_{\mathrm{obs}}}{\langle\sigma_{\mathrm{semi}} v_{\mathrm{rel}}\rangle_{\mathrm{fo}}}\right) \quad (18)$$

is not simplified, unlike the case where $\Delta m > 0$, although we can check whether an observational constraint is satisfied in a case-by-case manner. The left-hand side is a monotonically decreasing function of $v_{\mathrm{rel,sat}}$ and takes a maximum value of $v_{\mathrm{rel,obs}}$ at $v_{\mathrm{rel,sat}} = 0$. As discussed above, when $\Delta m = 0$, the observational constraints are satisfied so far, and thus no lower bound on $|\Delta m|$ is implied. Nevertheless, the mass difference should be maintained at a $\sim\mathcal{O}(1)\%$ level at maximum to avoid the Boltzmann suppression during the freeze-out.

V. CONCLUSION

In this paper, we presented a novel realization of SIMP as SIDM, where DM pions are associated with an ALP. Our model evaded shortcomings in the original SIMP model, such as an implicitly assumed mechanism of the kinetic equilibration between the pions and the SM thermal plasma and only marginal perturbativity. The former is solved by the ALP connecting the pions and the SM sector when the ALP mass is degenerate with the pion mass. Meanwhile, the latter is alleviated because the chemical equilibration receives a contribution from the semiannihilation in addition to the $3 \to 2$ process and the semiannihilation decouples later.

The newly introduced ALP is severely constrained by beam dump experiments. However, we have shown that in a viable parameter region, DM pions possess a sizable self-scattering cross section, which (at least partially) solves the small-scale crisis. Remarkably, most of the corresponding parameter region is within the reach of future beam dump experiments such as the SHiP experiment and will also be potentially probed by future cosmic gamma-ray searches such as e-ASTROGAM.

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APPENDIX: GENERALIZATION TO OTHER SYMMETRY GROUPS

In the main text, we consider N_f vectorlike fermion pairs that transform as the fundamental and antifundamental representations of a $\mathrm{SU}(N_c)$ gauge group. The model respects the global $\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R$ symmetry, except for a mass term that breaks its $\mathrm{SU}(N_f)_A$ subgroup explicitly. When the chiral condensation forms, the global symmetry is broken into its $\mathrm{SU}(N_f)_V$ subgroup, which is assumed to be an exact symmetry ensuring the longevity of the resultant pions. Our discussion does not change qualitatively for other gauge groups: (A) $\mathrm{SO}(N_c)$ ($N_c \geq 4$) and (B) $\mathrm{USp}(N_c)$ ($N_c \geq 4$).[2] We can introduce N_f copies of Weyl fermions that transform as the fundamental representation of the gauge group. Note that N_f should be even for the $\mathrm{USp}(N_c)$ gauge group so that the gauge group evades the global anomaly [72]. The Lagrangian density is invariant under $\mathrm{SU}(N_f)$, while a mass term is introduced so that only the following subgroup is respected, (A) $\mathrm{SO}(N_f)$ and (B) $\mathrm{USp}(N_f)$. In such a model, confinement and chiral condensation are expected to occur, and thus the

[2] In our notation, $\mathrm{USp}(2) \cong \mathrm{SU}(2)$ and thus N_c should be even. The skew-symmetric matrix is denoted by Ω.

TABLE I. Group factors in flavor-averaged cross sections of pions [Eqs. (7)–(10)] residing in different quotient spaces (G/H).

G/H	N_π	d^2
$SU(N_f)_L \times SU(N_f)_R/SU(N_f)_V$	$N_f^2 - 1$	$4(N_f^2-4)(N_f^2-1)/N_f$
$SU(N_f)/SO(N_f)$	$(N_f-1)(N_f+2)/2$	$(N_f-1)(N_f+4)(N_f^2-4)/N_f$
$SU(N_f)/USp(N_f)$	$(N_f-2)(N_f+1)/2$	$(N_f-4)(N_f+1)(N_f^2-4)/N_f$

$\{C+R\}^2$	$\{C+R\}D^2$
$8(N_f^2-1)(3N_f^4-2N_f^2+6)/N_f^2$	$16(N_f^2-4)(N_f^2-1)(N_f^2+10)/(3N_f^2)$
$(N_f-1)(N_f+2)(3N_f^4+7N_f^3-2N_f^2-12N_f+24)/N_f^2$	$2(N_f-1)(N_f+4)(N_f^2-4)(N_f^2-5N_f+20)/(3N_f^2)$
$(N_f-2)(N_f+1)(3N_f^4-7N_f^3-2N_f^2+12N_f+24)/N_f^2$	$2(N_f-4)(N_f+1)(N_f^2-4)(N_f^2+5N_f+20)/(3N_f^2)$

D^4	t^2
$32(11N_f^2-56)(N_f^2-4)(N_f^2-1)/(9N_f^2)$	$4(N_f^2-4)(N_f^2-1)N_f/3$
$4(N_f-1)(N_f+2)(11N_f^2+25N_f-112)(N_f^2-4)/(9N_f^2)$	$(N_f^2-4)(N_f^2-1)N_f/12$
$4(N_f-2)(N_f+1)(11N_f^2-25N_f-112)(N_f^2-4)/(9N_f^2)$	$(N_f^2-4)(N_f^2-1)N_f/12$

$\{CD\}^2$
$2(N_f^2-1)(N_f^2-4)(833N_f^4-6630N_f^2+11682)/(27N_f^3)$
$(N_f-1)(N_f+4)(N_f^2-4)(833N_f^4+3381N_f^3-10614N_f^2-20484N_f+46728)/(216N_f^3)$
$(N_f-4)(N_f+1)(N_f^2-4)(833N_f^4-3381N_f^3-10614N_f^2+20484N_f+46728)/(216N_f^3)$

low-energy theory can be described by a nonlinear sigma model where pions reside in (A) $SU(N_f)/SO(N_f)$ and (B) $SU(N_f)/USp(N_f)$ [73]. The effective Lagrangian density is similar to Eq. (4), while the broken $SU(N_f)$ generators should satisfy (A) $T^a = (T^a)^T$ and (B) $T^a\Omega = \Omega(T^a)^T$. It follows that the group factors in flavor-averaged cross sections [see Eqs. (7)–(10)], which are defined as follows, are different from one model to another, as summarized in Table I:

$$d^2 = \sum_{abc} d_{abc}^2, \tag{A1}$$

$$\{C+R\}^2 = \sum_{abcd}\{C+R\}_{abcd}^2, \\ = \sum_{abcd} c_{(abcd)}^2 + 16 r_{(ab)(cd)}^2/9, \tag{A2}$$

$$\{C+R\}D^2 = \sum_{abcd}\{C+R\}_{abcd} D_{abcd}^2, \tag{A3}$$

$$D^4 = \sum_{abcd}(D_{abcd}^2)^2, \tag{A4}$$

$$t^2 = \sum_{abcde} c_{[abcde]}^2, \tag{A5}$$

$$\{CD\}^2 = \sum_{abcde}\{CD\}_{abcde}^2, \tag{A6}$$

where we define

$$d_{abc} = \mathrm{Tr}(T^a\{T^b, T^c\})/2, \tag{A7}$$

$$r_{abcd} = \sum_e f_{ace} f_{bde}, \tag{A8}$$

$$\{C+R\}_{abcd} = c_{(abcd)} + 4r_{(ab)(cd)}/3, \tag{A9}$$

$$c_{abcd} = \mathrm{Tr}(T^aT^bT^cT^d), \tag{A10}$$

$$c_{abcde} = \mathrm{Tr}(T^aT^bT^cT^dT^e), \tag{A11}$$

$$D_{abcd}^2 = \sum_e [d_{abe}d_{cde}/3 \\ -(d_{ace}d_{bde} + d_{ade}d_{bce})], \tag{A12}$$

$$\{CD\}_{abcde} = 4c_{(abcde)} + \sum_f [c_{(abcf)}d_{def}/4 \\ - (c_{(abdf)}d_{cef} + c_{(acdf)}d_{bef} \\ + c_{(bcdf)}d_{aef} + c_{(abef)}d_{cdf} \\ + c_{(acef)}d_{bdf} + c_{(bcef)}d_{adf}) \\ + 2(d_{abf}c_{(cdef)} + d_{acf}c_{(bdef)} \\ + d_{bcf}c_{(adef)})/3], \tag{A13}$$

with f_{abc} being the structure coefficients of the broken generators.

Another caveat should be taken into account for the WZW term. The prefactor of the WZW term [$2N_c$ in Eq. (5)] is determined to reproduce the quantum anomaly of the fermion flavor symmetry [28,29], which we denote as $2k$. There is a factor of 2 in it in the case of the $\mathrm{SU}(N_c)$ gauge group since both N and $\bar{N}$ contribute to the quantum anomaly. We find that it is $2k = N_c$ in the other gauge groups. The WZW term can be written as an action on a five-dimensional ball, the boundary of which is the four-dimensional Minkowski spacetime,

$$\Gamma_{\mathrm{WZW}} = 2\pi k\nu, \quad \nu = -i\frac{1}{480\pi^3}\int \mathrm{Tr}(U^{-1}dU)^5, \qquad \text{(A14)}$$

with d being the exterior derivative. U is parametrized by pions as in Eq. (2) and given by (A) VV^T and (B) $V\Omega V^T\Omega^T$ with V being $\mathrm{SU}(N_f)$-valued fields. For the WZW term to be independent of a choice of a five-dimensional ball and thus well defined, the above WZW action on a five-dimensional sphere should be a multiple of 2π. In fact, ν on a five-dimensional sphere measures the winding number of U and thus takes an integer value. Note that k takes a half-integer value ($N_c/2$) for the $\mathrm{SO}(N_c)$ gauge group, while it is integer ($N_c/2$, but N_c is even) for the $\mathrm{USp}(N_c)$ gauge groups. The WZW term for the $\mathrm{SO}(N_c)$ gauge group with an odd N_c is, on the other hand, well defined since the winding number of $U = VV^T$ is twice that of V.[3]

[3] A similar discussion is found in Ref. [74].

[1] G. Bertone and D. Hooper, arXiv:1605.04909 [Rev. Mod. Phys. (to be published)].
[2] G. Jungman, M. Kamionkowski, and K. Griest, Phys. Rep. **267**, 195 (1996).
[3] G. Arcadi, M. Dutra, P. Ghosh, M. Lindner, Y. Mambrini, M. Pierre, S. Profumo, and F. S. Queiroz, arXiv:1703.07364.
[4] A. Del Popolo and M. Le Delliou, Galaxies **5**, 17 (2017).
[5] W. J. G. de Blok, Adv. Astron. **2010**, 789293 (2010).
[6] J. F. Navarro, C. S. Frenk, and S. D. M. White, Astrophys. J. **490**, 493 (1997).
[7] W. J. G. de Blok, F. Walter, E. Brinks, C. Trachternach, S. H. Oh, and R. C. Kennicutt, Jr., Astron. J. **136**, 2648 (2008).
[8] S. H. Oh, W. J. G. de Blok, E. Brinks, F. Walter, and R. C. Kennicutt, Astron. J. **141**, 193 (2011).
[9] S. H. Oh *et al.*, Astron. J. **149**, 180 (2015).
[10] K. A. Oman *et al.*, Mon. Not. R. Astron. Soc. **452**, 3650 (2015).
[11] D. N. Spergel and P. J. Steinhardt, Phys. Rev. Lett. **84**, 3760 (2000).
[12] N. Yoshida, V. Springel, S. D. M. White, and G. Tormen, Astrophys. J. **544**, L87 (2000).
[13] M. Vogelsberger, J. Zavala, and A. Loeb, Mon. Not. R. Astron. Soc. **423**, 3740 (2012).
[14] M. Rocha, A. H. G. Peter, J. S. Bullock, M. Kaplinghat, S. Garrison-Kimmel, J. Onorbe, and L. A. Moustakas, Mon. Not. R. Astron. Soc. **430**, 81 (2013).
[15] M. Vogelsberger, J. Zavala, F. Y. Cyr-Racine, C. Pfrommer, T. Bringmann, and K. Sigurdson, Mon. Not. R. Astron. Soc. **460**, 1399 (2016).
[16] A. Kamada, M. Kaplinghat, A. B. Pace, and H. B. Yu, arXiv:1611.02716.
[17] P. Creasey, O. Sameie, L. V. Sales, H. B. Yu, M. Vogelsberger, and J. Zavala, Mon. Not. R. Astron. Soc. **468**, 2283 (2017).
[18] M. Kaplinghat, R. E. Keeley, T. Linden, and H. B. Yu, Phys. Rev. Lett. **113**, 021302 (2014).
[19] O. D. Elbert, J. S. Bullock, M. Kaplinghat, S. Garrison-Kimmel, A. S. Graus, and M. Rocha, arXiv:1609.08626.
[20] A. H. G. Peter, M. Rocha, J. S. Bullock, and M. Kaplinghat, Mon. Not. R. Astron. Soc. **430**, 105 (2013).
[21] M. Markevitch, A. H. Gonzalez, D. Clowe, A. Vikhlinin, W. Forman, C. Jones, S. Murray, and W. Tucker, Astrophys. J. **606**, 819 (2004).
[22] S. W. Randall, M. Markevitch, D. Clowe, A. H. Gonzalez, and M. Bradac, Astrophys. J. **679**, 1173 (2008).
[23] M. Kaplinghat, S. Tulin, and H. B. Yu, Phys. Rev. Lett. **116**, 041302 (2016).
[24] R. Massey *et al.*, Mon. Not. R. Astron. Soc. **449**, 3393 (2015).
[25] F. Kahlhoefer, K. Schmidt-Hoberg, J. Kummer, and S. Sarkar, Mon. Not. R. Astron. Soc. **452**, L54 (2015).
[26] Y. Hochberg, E. Kuflik, T. Volansky, and J. G. Wacker, Phys. Rev. Lett. **113**, 171301 (2014).
[27] Y. Hochberg, E. Kuflik, H. Murayama, T. Volansky, and J. G. Wacker, Phys. Rev. Lett. **115**, 021301 (2015).
[28] J. Wess and B. Zumino, Phys. Lett. **37B**, 95 (1971).
[29] E. Witten, Nucl. Phys. **B223**, 422 (1983).
[30] E. D. Carlson, M. E. Machacek, and L. J. Hall, Astrophys. J. **398**, 43 (1992).
[31] H. M. Lee and M. S. Seo, Phys. Lett. B **748**, 316 (2015).
[32] Y. Hochberg, E. Kuflik, and H. Murayama, J. High Energy Phys. 05 (2016) 090.
[33] A. Kamada, M. Yamada, T. T. Yanagida, and K. Yonekura, Phys. Rev. D **94**, 055035 (2016).
[34] A. Manohar and H. Georgi, Nucl. Phys. **B234**, 189 (1984).
[35] H. Georgi, Phys. Lett. B **298**, 187 (1993).
[36] M. Hansen, K. Langæble, and F. Sannino, Phys. Rev. D **92**, 075036 (2015).
[37] S. M. Choi and H. M. Lee, J. High Energy Phys. 09 (2015) 063.
[38] S. M. Choi and H. M. Lee, Phys. Lett. B **758**, 47 (2016).

[39] S. M. Choi, Y. J. Kang, and H. M. Lee, J. High Energy Phys. 12 (2016) 099.

[40] S. M. Choi, H. M. Lee, and M. S. Seo, J. High Energy Phys. 04 (2017) 154.

[41] F. D'Eramo and J. Thaler, J. High Energy Phys. 06 (2010) 109.

[42] P. Di Vecchia and G. Veneziano, Nucl. Phys. **B171**, 253 (1980).

[43] E. Witten, Ann. Phys. (N.Y.) **128**, 363 (1980).

[44] A. Kamada, H. J. Kim, H. Kim, and T. Sekiguchi (to be published).

[45] E. Izaguirre, T. Lin, and B. Shuve, Phys. Rev. Lett. **118**, 111802 (2017).

[46] F. Bergsma *et al.* (CHARM Collaboration), Phys. Lett. **157B**, 458 (1985).

[47] J. Blumlein *et al.*, Z. Phys. C **51**, 341 (1991).

[48] J. D. Bjorken, S. Ecklund, W. R. Nelson, A. Abashian, C. Church, B. Lu, L. W. Mo, T. A. Nunamaker, and P. Rassmann, Phys. Rev. D **38**, 3375 (1988).

[49] E. M. Riordan *et al.*, Phys. Rev. Lett. **59**, 755 (1987).

[50] M. Anelli *et al.* (SHiP Collaboration), arXiv:1504.04956.

[51] B. Döbrich, J. Jaeckel, F. Kahlhoefer, A. Ringwald, and K. Schmidt-Hoberg, J. High Energy Phys. 02 (2016) 018.

[52] T. Abe *et al.* (Belle-II Collaboration), arXiv:1011.0352.

[53] J. E. Kim, Phys. Rev. Lett. **43**, 103 (1979).

[54] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B166**, 493 (1980).

[55] J. Jaeckel and M. Spannowsky, Phys. Lett. B **753**, 482 (2016).

[56] T. Behnke *et al.*, arXiv:1306.6327.

[57] M. Bicer *et al.* (TLEP Design Study Working Group Collaboration), J. High Energy Phys. 01 (2014) 164.

[58] V. Khachatryan *et al.* (CMS Collaboration), Phys. Rev. D **94**, 112004 (2016).

[59] S. Chatrchyan *et al.* (CMS Collaboration), Phys. Rev. D **87**, 092008 (2013).

[60] E. Masso and R. Toldra, Phys. Rev. D **52**, 1755 (1995).

[61] L. Bergström, Rep. Prog. Phys. **63**, 793 (2000).

[62] X. L. Chen and M. Kamionkowski, Phys. Rev. D **70**, 043502 (2004).

[63] N. Padmanabhan and D. P. Finkbeiner, Phys. Rev. D **72**, 023508 (2005).

[64] P. A. R. Ade *et al.* (Planck Collaboration), Astron. Astrophys. **594**, A13 (2016).

[65] T. R. Slatyer, Phys. Rev. D **93**, 023527 (2016).

[66] M. Kawasaki, K. Nakayama, and T. Sekiguchi, Phys. Lett. B **756**, 212 (2016).

[67] A. R. Pullen, R. R. Chary, and M. Kamionkowski, Phys. Rev. D **76**, 063006 (2007); **83**, 029904(E) (2011).

[68] M. Ackermann *et al.* (Fermi-LAT Collaboration), Phys. Rev. D **91**, 122002 (2015).

[69] G. Battaglia, A. Helmi, H. Morrison, P. Harding, E. W. Olszewski, M. Mateo, K. C. Freeman, J. Norris, and S. A. Shectman, Mon. Not. R. Astron. Soc. **364**, 433 (2005); **370**, 1055(E) (2006).

[70] A. De Angelis *et al.* (e-ASTROGAM Collaboration), Exp. Astron. 1 (2017).

[71] R. T. D'Agnolo and J. T. Ruderman, Phys. Rev. Lett. **115**, 061301 (2015).

[72] E. Witten, Phys. Lett. **117B**, 324 (1982).

[73] E. Witten, Nucl. Phys. **B223**, 433 (1983).

[74] R. Auzzi, S. Bolognesi, and M. Shifman, Phys. Rev. D **77**, 125029 (2008).