

## Collisional Dynamics of Half-Quantum Vortices in a Spinor Bose-Einstein Condensate

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We present an experimental study on the interaction and dynamics of half-quantum vortices (HQVs) in an antiferromagnetic spinor Bose-Einstein condensate. By exploiting the orbit motion of a vortex dipole in a trapped condensate, we perform a collision experiment of two HQV pairs, and observe that the scattering motions of the HQVs is consistent with the short-range vortex interaction that arises from nonsingular magnetized vortex cores. We also investigate the relaxation dynamics of turbulent condensates containing many HQVs, and demonstrate that spin wave excitations are generated by the collisional motions of the HQVs. The short-range vortex interaction and the HQV-magnon coupling represent two characteristics of the HQV dynamics in the spinor superfluid.

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When a superfluid has an internal spin degree of freedom, quantum vortices (QVs) with fractional values of the quantum circulation,  $h/m$ , may exist in the superfluid due to the complex topology of the order parameter manifold [1,2], where  $h$  is Planck's constant and  $m$  is particle mass. Recently, QVs with  $h/2m$ , known as half-quantum vortices (HQVs), were reported with many spinor superfluid systems;  $\pi$  rotation of light polarization was shown around defects in exciton-polariton condensates [3,4], half-height magnetization steps were detected in a ring-shaped spin-triplet superconductor of  $\text{Sr}_2\text{RuO}_4$  [5], and spontaneous dissociation of a singly charged vortex into a pair of HQVs with ferromagnetic cores was observed in an antiferromagnetic spinor Bose-Einstein condensate (BEC) [6]. More recently, HQVs were identified in NMR measurements of a rotating superfluid  $^3\text{He}$  in the polar phase [7].

Given the stable existence of HQVs in the spinor superfluids, the next immediate questions concern their dynamic properties [8–12]. Since a HQV intrinsically involves a spin texture, the interplay between mass and spin flows will play an important role in the HQV dynamics. In particular, a HQV has a nonsingular core structure where the core region is occupied by a nonrotating spin component, thus, having a continuous vorticity distribution. This is in stark contrast to the conventional QV which has a density-vanishing core with phase singularity. Therefore, the HQV dynamics would be qualitatively different from that of QVs in a scalar superfluid. It has been anticipated that HQVs can merge into and come out of topological solitons such as Skrymions and merons [13,14], and spin monopoles [15–17].

In this Letter, we present an experimental study on the dynamics of HQVs in an antiferromagnetic spinor BEC. By means of a vortex-dipole generation technique, we perform a collision experiment of two HQV pairs in a highly oblate BEC and show that two HQVs with opposite core

magnetizations have a short-range interaction. We also investigate the relaxation dynamics of turbulent BECs containing many HQVs and observe that spin wave excitations are generated by the collisional motions of HQVs, revealing the dissipative mechanism in the HQV dynamics. The short-range interaction of HQVs and the coupling between HQVs and magnons represent the prominent features of the HQV dynamics. Our findings manifest the dynamic interplay of the mass and spin sectors in the spinor superfluid system.

We study a BEC of  $^{23}\text{Na}$  atoms in the  $F = 1$  hyperfine spin state. The spin interaction in the spinor condensate is antiferromagnetic, whose energy is given as  $E_S = (c_2 n/2) \langle \mathbf{F} \rangle^2$  with  $c_2 > 0$ , where  $n$  is the atomic density and  $\mathbf{F} = (F_x, F_y, F_z)$  is the single-particle spin operator. The ground spin state is polar with  $\langle \mathbf{F} \rangle = 0$  that can be parametrized with a unit vector  $\vec{d}$ , such that the system is in the  $m_F = 0$  spin state for the quantization axis along  $\vec{d}$  [18,19]. The order parameter of the condensate is expressed as

$$\Psi = \begin{pmatrix} \psi_{+1} \\ \psi_0 \\ \psi_{-1} \end{pmatrix} = \sqrt{n} e^{i\theta} \begin{pmatrix} \frac{-d_x + id_y}{\sqrt{2}} \\ d_z \\ \frac{d_x + id_y}{\sqrt{2}} \end{pmatrix}, \quad (1)$$

where  $\psi_l$  is the  $m_z = l$  spin component along the  $z$  direction ( $l = 0, \pm 1$ ) and  $\theta$  is the superfluid phase. The order parameter is invariant under  $\theta \rightarrow \theta + \pi$  and  $\vec{d} \rightarrow -\vec{d}$ . This  $Z_2$  symmetry allows HQV formation consisting of  $\pi$  phase winding together with spin flipping around the vortex core [20–22].

In the presence of an external magnetic field, e.g., along the  $z$  direction, the quadratic Zeeman shift,  $q$ , introduces a uniaxial spin anisotropy to the system with the energy of  $E_Z = q \langle F_z^2 \rangle = q(1 - d_z^2)$ . The ground state has a spin

orientation  $\vec{d} \parallel \hat{z}$  for  $q > 0$  and  $\vec{d} \perp \hat{z}$  for  $q < 0$ . We refer to these two phases as the easy-axis polar (EAP) and easy-plane polar (EPP) phases, respectively. In the EPP phase, the spin can rotate in the  $xy$  plane, admitting formation of HQVs having two topological charges,  $q_n$  and  $q_s$  ( $|q_n| = |q_s| = \frac{1}{2}$ ), which are the winding numbers of  $\theta$  and  $\vec{d}$ , respectively. The EPP phase is an equal mixture of the  $m_z = +1$  and  $-1$  spin components, and HQV can be understood as a quantum vortex in one spin component whose core is occupied by the other nonrotating spin component and, thus, is magnetized. The characteristic size of the ferromagnetic core is given by the spin healing length  $\xi_s = \hbar/\sqrt{2mc_2n}$  [8,10].

The interaction between vortices arises mainly from the interference of the velocity fields generated by the vortices. Much like for QVs in a scalar superfluid, the HQV-HQV interaction energy shows a logarithmic form of  $\kappa \ln R$  for a large distance  $R \gg \xi_s$ , where  $\kappa = q_{n1}q_{n2} + q_{s1}q_{s2}$ , parametrizing the overall interference effect in the mass and spin currents, and  $(q_{ni}, q_{si})$  are the charges of the  $i$ th HQV ( $i = 1, 2$ ). When  $R$  is comparable to the core size  $\xi_s$ , the core structure is modified in the proximity of the other vortex, consequently affecting the HQV interaction. Particularly, in the case of  $\kappa = 0$  where the two HQVs have different core magnetizations (Fig. 1), the core deformation is the main mechanism governing the HQV

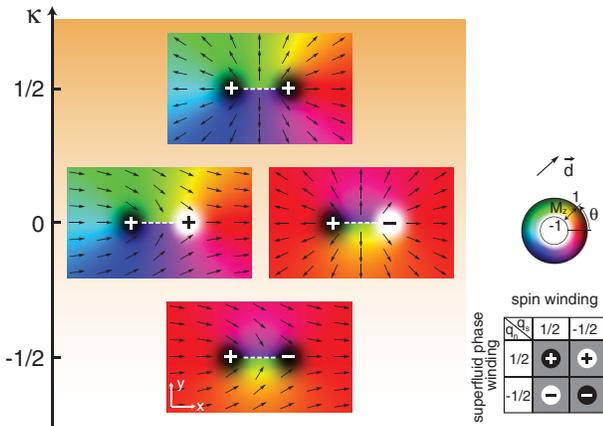


FIG. 1. HQV pairs in an antiferromagnetic spinor BEC in the easy-plane polar (EPP) phase. The diagrams show the spatial structures of four HQV pairs in terms of the superfluid phase  $\theta$  (color) and the spin orientation  $\vec{d}$  (arrow). The magnetization  $M_z$  in the core region is indicated by brightness. The topological charges of the left and right HQVs are  $(q_{n1}, q_{s1}) = (\frac{1}{2}, \frac{1}{2})$  and  $(|q_{n2}|, |q_{s2}|) = (\frac{1}{2}, \frac{1}{2})$ , respectively, where  $q_n$  and  $q_s$  correspond to the winding numbers of  $\theta$  and  $\vec{d}$ , respectively.  $\kappa = q_{n1}q_{n2} + q_{s1}q_{s2}$ . The dashed line between the two HQVs denotes a sudden change of  $\theta \rightarrow \theta + \pi$  and  $\vec{d} \rightarrow -\vec{d}$ . The superfluid order parameter is continuous across the dashed line due to the  $Z_2$  symmetry (see text). The HQV pair with  $\kappa = -1/2$  is a HQV dipole with zero net charges.

interaction. Theoretical studies predicted that the HQV interaction for  $\kappa = 0$  is repulsive and short-ranged with an asymptotic form of  $(\ln R)/R^2$  [8,10].

Our experiment starts by preparing a nearly pure BEC of  $^{23}\text{Na}$  atoms in the  $|F = 1, m_F = 0\rangle$  hyperfine spin state in an oblate optical dipole trap [6,23]. The condensate typically contains  $N_a = 5.6 \times 10^6$  atoms with Thomas-Fermi radii  $(R_x, R_y, R_z) \approx (203, 163, 1.8) \mu\text{m}$ . For peak atom density, the density and spin healing lengths are  $\xi_n \approx 0.5 \mu\text{m}$  and  $\xi_s \approx 4.1 \mu\text{m}$ , respectively. Since  $\xi_s > R_z$ , the spin dynamics in the oblate condensate is effectively two dimensional. The external magnetic field is  $B_z = 30 \text{ mG}$  along the  $z$  direction, giving  $q/h = 0.24 \text{ Hz}$ . The value of  $q$  can be tuned to be negative by using a microwave dressing technique [25–27]. HQVs are detected as ferromagnetic

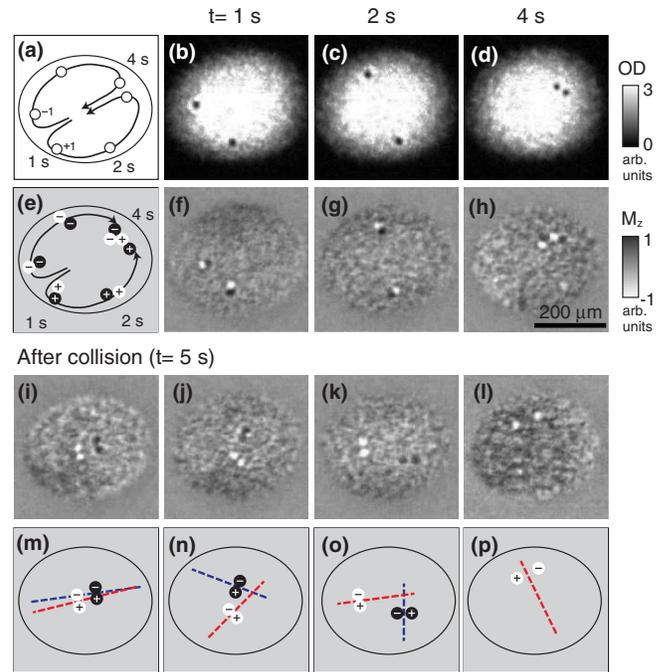


FIG. 2. Collision of HQV pairs. (a) A vortex dipole consisting of two singly charged vortices with opposite circulations is generated in the center region of the condensate [23]. The two vortices show an orbit motion depicted by the solid lines. (b)–(d) Optical density (OD) images of BECs in the easy-axis polar (EAP) phase, taken at (b)  $t = 1 \text{ s}$ , (c)  $2 \text{ s}$ , and (d)  $4 \text{ s}$  after generating a vortex dipole. (e) When the BEC is transmuted to the EPP phase after the vortex generation, each singly charged vortex is split into a pair of HQVs with opposite core magnetizations, and the HQV pair moves along the trajectory of its original singly charged vortex. (f)–(h) Magnetization images of BECs in the EPP phase at (f)  $t = 1 \text{ s}$ , (g)  $2 \text{ s}$ , (h)  $4 \text{ s}$ , and (i)–(l)  $5 \text{ s}$ . At  $t \approx 4 \text{ s}$ , two HQV pairs collide in the upper right region of the BEC and scatter into two HQV dipoles. (m)–(p) Descriptions of the vortex states in (i)–(l). The dashed lines indicate the propagation lines of the HQV dipoles. In (l), only one HQV dipole is identified. All images were taken after a free expansion of the samples for 24 ms.

point defects in magnetization imaging [6,23,28], or with density-depleted cores in absorption imaging after Stern-Gerlach spin separation.

We perform a collision experiment of HQVs by exploiting the orbit motion of a vortex dipole in a trapped condensate [Fig. 2(a)] [29,30]. First, we generate a vortex dipole of two singly charged vortices with opposite circulations by sweeping the center region of the condensate with a penetrable repulsive laser beam [23,31]. The vortex dipole linearly propagates, and near the condensate boundary, the two vortices separate and circulate in opposite directions along the condensate perimeter [Figs. 2(b) and 2(c)]. They eventually combine at the other side of the condensate [Fig. 2(d)] and traverse the condensate again as a small dipole. This orbit motion can be explained with the velocity field of the vortices and the boundary condition of the trapped condensate [29,30]. In our sample, the orbit motion period was about 4.5 s.

For a HQV collision experiment, we transmute the condensate from the EAP phase into the EPP phase immediately after the vortex generation. It is achieved by applying a  $\pi/2$  rf pulse to rotate  $\hat{d}$  from  $\hat{z}$  to the  $xy$  plane and, subsequently, changing  $q/h$  to  $-10$  Hz. In the EPP phase, a singly charged vortex with  $(q_n, q_s) = (\pm 1, 0)$  is unstable and dissociated into two HQVs with  $(q_n, q_s) = (\pm \frac{1}{2}, \frac{1}{2})$  and  $(\pm \frac{1}{2}, -\frac{1}{2})$  [6]. We observe that the splitting

process is completed within 1.5 s and the two HQV pairs move along the trajectories of their original singly charged vortices, maintaining their small pair separation [Figs. 2(f) and 2(g)]. Note that, thanks to the vortex orbit motion and the core magnetization, we can unambiguously specify the charges  $(q_n, q_s)$  of each HQV [Fig. 2(e)]. The pair separation was about  $4\xi_s$  and its direction was random in each realization of the experiment [6]. It was predicted that a HQV pair created from a singly charged vortex undergoes pair rotation due to its repulsive short-range interaction [11,12].

The two HQV pairs collide in the upper right region of the condensate [Fig. 2(h)] and scatter into two HQV dipoles with zero net charges: one is a  $\{(\frac{1}{2}, \frac{1}{2}), (-\frac{1}{2}, -\frac{1}{2})\}$  dipole with spin-up core magnetization and the other is a  $\{(\frac{1}{2}, -\frac{1}{2}), (-\frac{1}{2}, +\frac{1}{2})\}$  dipole with spin-down core magnetization [Figs. 2(i)–2(k)]. We see that the propagation lines of the two HQV dipoles cross in the region where the four HQVs gathered [Figs. 2(m)–2(o)], recalling the HQV collision event. The observed splitting, orbiting, and scattering motions of the HQVs corroborate the existence of the short-range interaction between HQVs with different core magnetizations.

Although in as few as 4 out of about 130 runs, we made an interesting observation where only one HQV dipole remains after collision, as shown in Fig. 2(l). This

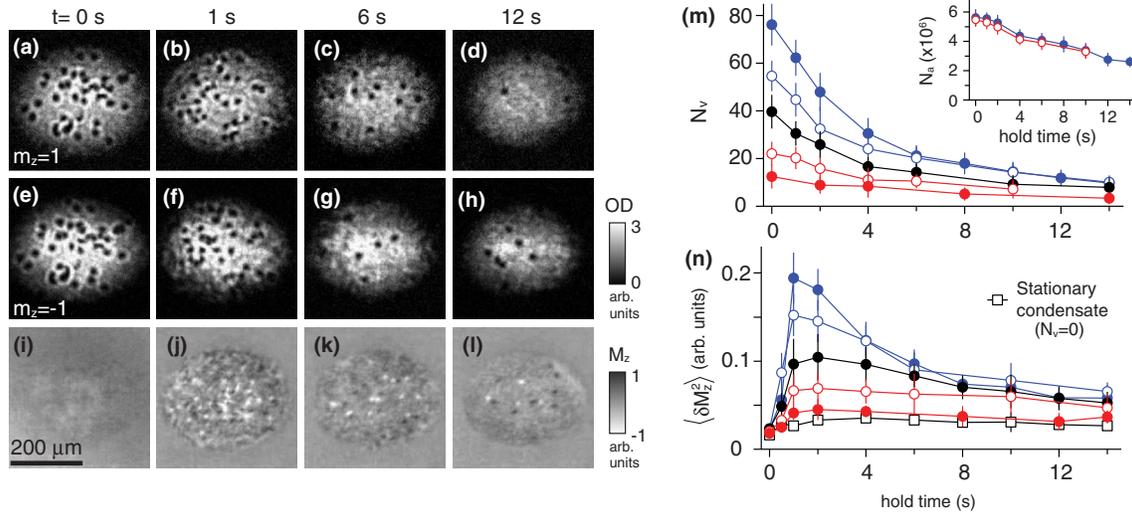


FIG. 3. Relaxation of quantum turbulence with HQVs. OD images of (a)–(d) the  $m_z = 1$  and (e)–(h)  $m_z = -1$  spin components of turbulent BECs at various hold times,  $t$ , in the EPP phase. The two spin components were imaged simultaneously with Stern-Gerlach spin separation and 24 ms time of flight [23]. The vortex positions in the two spin components coincide initially and become uncorrelated in the subsequent evolution, indicating the development of HQVs. The cloud shapes of the two spin components are slightly mismatched due to the inhomogeneity of the spin-separating field gradient. (i)–(l) *In situ* magnetization images of the BECs for the corresponding hold times. Spin wave excitations are rapidly generated as HQVs develop. Temporal evolutions of (m) the HQV number,  $N_v$ , and (n) the variance of spin fluctuations,  $\langle \delta M_z^2 \rangle$ , for various sample conditions.  $N_v$  was measured from the OD images of the two spin components and  $\langle \delta M_z^2 \rangle$  was determined from the central  $150 \mu\text{m} \times 150 \mu\text{m}$  region of the *in situ* magnetization image. The data points were obtained by averaging ten measurements of the same experiment, and the error bars indicate the standard deviation of the measurements. The inset in (m) shows the atom number evolution of the BEC for initial vortex number  $N_v \approx 80$  (solid) and 20 (open). Over 12 s evolution, the condensate fraction slightly decreased from  $> 90\%$  to  $\approx 85\%$ .

implies that the other HQV dipole was annihilated during the collision process. The pair annihilation requires energy dissipation, which possibly happens via dynamic coupling of HQVs to other excitations in the system, such as phonons and magnons. In the EPP phase, there are two modes of spin excitation: the gapless axial magnons due to the broken spin rotation symmetry in the  $xy$  plane, and the gapped transverse magnons associated with the  $m_z = 0$  spin component [21]. The ferromagnetic cores of a HQV dipole would be mainly released as axial magnons in the annihilation process. In recent numerical simulations of the HQV dipole dynamics based on 2D Gross-Pitaevskii equations, a peculiar tendency of pair annihilation was observed for short intervortex separation [12].

To investigate the dissipative nature of the HQV dynamics, we examine the relaxation of a turbulent BEC containing many HQVs (Fig. 3). We prepare a condensate with a spatially disordered vortex distribution in the EAP phase [23,32] and transmute it into the EPP phase. Large spin wave excitations are rapidly generated as the singly charged vortices are dissociated and the resultant HQVs are spatially scrambled [Figs. 3(i) and 3(j)]. Then, the turbulent condensate gradually relaxes to a stationary state, decreasing the HQV number,  $N_v$ . We characterize the relaxation dynamics with the temporal evolutions of  $N_v$  and the variance of spatial fluctuations,  $\langle \delta M_z^2 \rangle$  [Figs. 3(m) and 3(n)] [23].  $\langle \delta M_z^2 \rangle$  reflects the axial magnon population in the condensate [6,33]. We observed no  $m_z = 0$  spin component in the relaxation dynamics, excluding the involvement of the gapped transverse magnons.

In Figure 4, we display the relaxation trajectories of the turbulent condensate in the plane of the dimensionless HQV density,  $n_v \xi_s^2$ , and  $\langle \delta M_z^2 \rangle$ , where  $n_v = N_v / (\pi R_x R_y)$ . Interestingly, we see that the turbulent condensate follows a certain universal relaxation curve after it reaches its maximum value of  $\langle \delta M_z^2 \rangle$ , regardless of the initial vortex number. One plausible explanation is that the magnon population is dynamically governed by the rate of magnon generation from the collisional motions of the HQVs. Along the universal relaxation curve, the dependence of  $\langle \delta M_z^2 \rangle$  on  $n_v \xi_s^2$  appears faster than linear, which seems to support the explanation because the occurrence probability of HQV collision must increase faster than linearly with the vortex density.

Finally, we analyze the decay curve of  $N_v$  in the turbulent condensate (Fig. 4, inset). Nonexponential decay behavior is observed, where the decay rate is reduced from  $0.2 \text{ s}^{-1}$  to  $0.1 \text{ s}^{-1}$  as  $N_v$  decreases from 60 to lower than 10. For comparison, we measured the decay curve of the singly charged vortex number with the same turbulent condensates in the EAP phase, and found that it decays exponentially over the entire experimental range with a decay rate of about  $0.03 \text{ s}^{-1}$  [23]. The fast and nonexponential decay behavior in the EPP phase highlights the role of the

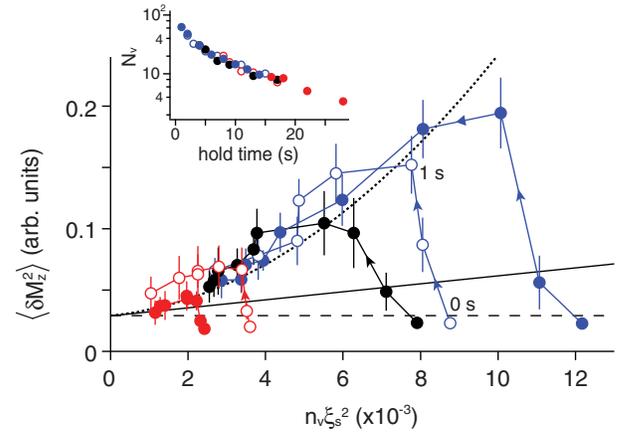


FIG. 4. Relaxation trajectories of turbulent BECs in the plane of the dimensionless HQV density,  $n_v \xi_s^2$ , and  $\langle \delta M_z^2 \rangle$ . The arrows represent the evolution direction of the trajectories. The dashed line indicates the thermal equilibrium value of  $\langle \delta M_z^2 \rangle$  for the stationary condensate, which are obtained by averaging the data points marked by open squares in Fig. 3(n) for long hold times  $t \geq 6 \text{ s}$ . The solid line shows the estimated contribution of magnetized HQV cores,  $\langle \delta M_z^2 \rangle_{\text{HQV}}$  [23], with the thermal equilibrium offset. The dotted line is a guide for the eyes. The variation of the atom number of the condensate was taken into account in the calculation of  $n_v \xi_s^2$  [Fig. 3(m), inset]. The inset shows the universal decay curve of the vortex number, constructed by joining the data in Fig. 3(m) for  $t \geq 1 \text{ s}$  with adjusted hold time offsets.

HQV-magnon coupling as a dissipative mechanism in the spinor superfluid system.

In conclusion, we investigated the dynamics of HQVs in the antiferromagnetic spinor Bose-Einstein condensate and demonstrated the existence of the short-range interaction of HQVs and the dynamic coupling between HQVs and magnons. It would be interesting to extend this work to the quantum critical point of  $q = 0$ , where the system recovers the full spin rotation symmetry of  $S^2$  and the spin winding number,  $q_s$ , is not topologically defined. Furthermore, in a 2D regime, no spin ordering would form at finite temperature [34,35], and it was anticipated that the so-called paired superfluid state might emerge near the critical point [36–38].

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