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Couplings between QCD axion and photon from string compactification

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ABSTRACT

The QCD axion couplings of various invisible axion models are presented. In particular, the exact global symmetry $U(1)_{PQ}$ in the superpotential is possible for the anomalous U(1) from string compactification, broken only by the gauge anomalies at one loop level, and is shown to have the resultant invisible axion coupling to photon, $c_{a\gamma\gamma} \geq \frac{8}{3} - c_{a\gamma\gamma}^{\rm ch\,br}$ where $c_{a\gamma\gamma}^{\rm ch\,br} \simeq 2$. However, this bound is not applicable in approximate $U(1)_{PQ}$ models with sufficiently suppressed $U(1)_{PQ}$-breaking superpotential terms. We also present a simple method to obtain $c_{a\gamma\gamma}^0$ which is the value obtained above the electroweak scale.

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1. Introduction

The detection possibility of the invisible axions [1–3] chiefly relies on its appreciable couplings to photon $c_{a\gamma\gamma}$ which appears in the Lagrangian as

$$\mathcal{L}_{\rm axion\ coupling} = -\frac{a}{32\pi^2 f_a}\left(c_3 g_3^2 G^a\tilde{G}^a + c_{a\gamma\gamma} e^2 F_{\rm em}\tilde{F}_{\rm em}\right), \qquad (1)$$

where

$$G^a\tilde{G}^a = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} G^a_{\mu\nu} G^a_{\rho\sigma}, \; F_{\rm em}\tilde{F}_{\rm em} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} F^{\mu\nu}_{\rm em}\tilde{F}^{\rho\sigma}_{\rm em}. \qquad (2)$$

The axionic domain-wall number [4] is $|c_3 + c_2|$ where c_2 is the contribution from the standard model quarks [5]. With this normalization from the QCD sector, the coupling $c_{a\gamma\gamma}$ is defined and is composed of two parts,

$$c_{a\gamma\gamma} = c^0_{a\gamma\gamma} + c^{\rm ch\,br}_{a\gamma\gamma} \simeq c^0_{a\gamma\gamma} - 2 \qquad (3)$$

where $c^0_{a\gamma\gamma}$ is the one obtained above the electroweak scale and $c^{\rm ch\,br}_{a\gamma\gamma}$ is the contribution obtained below the QCD chiral phase transition scale. Since the mass ratio of up and down quarks is very close to 0.5 [6], we use the value $m_u/m_d = 0.5$ below. In this case, $c^{\rm ch\,br}_{a\gamma\gamma}$ is -2 (a bit smaller value -1.98, including the strange quark contribution) [7]. The early summary on the axion–photon–photon couplings was summarized in [5,8].[1]

An invisible axion is a pseudo-Goldstone boson whose mother symmetry is the Peccei–Quinn (PQ) symmetry [10]. The symmetry breaking scale relevant for the axion detection experiments [11,12] is the intermediate scale 10^{10} GeV $\lesssim f_a \lesssim 10^{12}$ GeV, which can be achieved by the vacuum expectation value (VEV) of an $SU(2) \times U(1)_Y$ singlet σ [1]. But, the global symmetry which is broken at the intermediate scale has to be fine-tuned to avoid the gravity spoil of global symmetries. This difficulty has been appreciated [13] after realizing that even the classical gravity does not necessarily preserve global symmetries due to the topology change via wormholes [14,15]. The wormhole taking out gauge charges is depicted in Fig. 1. An observer in the almost flat space O notices that some gauge charges are flown to the shadow world S. To see the effect in his own space only, he cuts the wormhole, then notices that the escaped gauge charges are recovered to O. This conservation of gauge charges in the space O is due to the long range electric flux lines. For the global charges, there is no such flux lines and hence the escaped global charges are not considered

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[1] The earlier unification value was given in [9].

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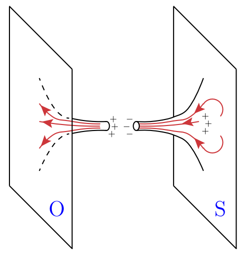

Fig. 1. Escaping charges through a wormhole.

to be recovered to O if he cuts the wormhole. Thus, global charges are broken if we consider the topology change. Related to this effect, at field theory level within supersymmetric (SUSY) framework, a host of discrete symmetries are considered [16–20]. Some discrete symmetries can lead to acceptable approximate global symmetries [21,22].

In this paper, we attempt to obtain a region of the parameter space from string compactified 4-dimensional (4D) effective field theory. The 4D models from string compactification do not allow global symmetries but allow some discrete symmetries [23]. The minimal supersymmetric standard model supplied with singlets σ (to house the invisible axion as pointed out in [1]) will be called σMSSM. In the σMSSM, we calculate the couplings between axion and photon.

2. Gauge transformation of model-independent axion

Four dimensional pseudoscalars in the σMSSM from string compactification appear from B_{MN} ($= 10D$ antisymmetric tensor field with $M, N \in \{1, 2, \cdots, 10\}$) and from the matter supermultiplets in the σMSSM. In 10D, B_{MN} is a gauge field satisfying the gauge transformation,

$$B_{MN} \to B_{MN} - \partial_M \Lambda_N + \partial_N \Lambda_M, \tag{4}$$

where Λ_M are gauge functions. If both M and N take the internal space coordinates $i, j = \{5, 6, \cdots, 10\}$, B_{ij} is a pseudoscalar in 4D. From the 4D point of view, the original gauge transformation is not local, not carrying the 4D index $\mu = \{1, 2, 3, 4\}$. These pseudoscalars are the model-dependent (MD) axion [24], which is known to generate superpotential terms [25]. So, the MD axions are not the useful candidates for solutions of the strong CP problem. On the other hand the model-independent (MI) axion [26,27], where both M and N of B_{MN} take 4D indices μ, ν, is a good candidate of 4D gauge transformation. Namely, the gauge transformation (4) is still a gauge transformation in 4D. So, $B_{\mu\nu}$ is not spoiled by gravity after the compactification.

2.1. Anomaly-free U(1) gauge symmetry

If the MI axion is not behaving as a longitudinal degree of a gauge boson, then the axion potential is generated and the bosonic collective motion behaves as cold dark matter (CDM) [12]. But, then the axion decay constant is near the string scale, $f_a > 10^{16}$ GeV [28], and a fine-tuning is needed, or the anthropic scenario must be invoked [29]. The coupling $c_{a\gamma\gamma}$ is the same as the one considered in the following subsection with anomalous U(1) without extra charged singlets.

2.2. Anomalous U(1) gauge symmetry

If the compactification produces an anomalous $\mathrm{U}(1)_{\rm anom}$ gauge symmetry, the corresponding $\mathrm{U}(1)_{\rm anom}$ gauge boson obtains a large mass, at a slightly lower scale than the string scale. The presence of a Fayet–Illiopoulos D-term connects the MI axion with the anomalous $\mathrm{U}(1)_{\rm anom}$ gauge boson [30], which is a kind of the Higgs mechanism providing the longitudinal degree of the gauge boson. The generator of the anomalous $\mathrm{U}(1)_{\rm anom}$ belongs to the $E_8 \times E_8$ algebra, and matter fields have the $\mathrm{U}(1)_{\rm anom}$ charges. The field $B_{\mu\nu}$ or the MI axion does not couple to matter fields. So below the $\mathrm{U}(1)_{\rm anom}$ gauge boson mass scale, the $\mathrm{U}(1)_{\rm anom}$ charge of matter fields becomes a global charge which can be called a $\mathrm{U}(1)_{\rm PQ}$ charge. In this way, a global symmetry free of the gravity obstruction is created below the $\mathrm{U}(1)_{\rm anom}$ gauge boson mass scale. Since the mother $\mathrm{U}(1)_{\rm anom}$ is a gauge symmetry, there is no gravity obstruction of this $\mathrm{U}(1)_{\rm PQ}$ global symmetry. In string compactification, it has been explicitly shown that the anomalies are the same for all gauge groups both for non-Abelian and (properly normalized) Abelian gauge fields [31,32]. However, this statement holds only for orbifold compactifications, where there is a single axion, whose shift must cancel all anomalies. In smooth compactifications one has many axion fields and therefore universality of anomalies does not hold anymore. There are such examples in the compactification of Type-I and Type-IIB string, where three anomalous U(1)'s were constructed [38]. Therefore, our present study has a limitation arising from the orbifold compactification of the heterotic string. Nevertheless, since the full phenomenologically acceptable σMSSM spectra of matter fields have been so far presented in the heterotic compactification, our study of anomalous U(1) gauge symmetries can be gates to low energy gravity-safe global symmetries. Now, we restrict to the case of $B_{\mu\nu}$ from the heterotic string.

Let the anomalous gauge symmetry be $\mathrm{U}(1)_{\rm anom}$. Its charge operator and the coupling constant be Γ and e_Γ, respectively. The potential which is invariant under $\mathrm{U}(1)_{\rm anom}$ is also invariant under the global symmetry $\mathrm{U}(1)_\Gamma$ whose charge generator is also Γ. To see the effective global symmetry below the anomalous scale, therefore, it is sufficient to see how the local transformation is described. Since the longitudinal degree of the $\mathrm{U}(1)_{\rm anom}$ gauge boson is solely provided by $B_{\mu\nu}$, matter scalars having the nonvanishing Γ charge do not develop VEVs. To see the $\mathrm{U}(1)_{\rm anom}$ gauge transformation of a complex scalar Φ, consider the kinetic energy term $(D^\mu\Phi)^*(D_\mu\Phi)$ where $D_\mu = \partial_\mu - ie_\Gamma \Gamma A_\mu$. The gauge transformation $\Phi \to e^{i\alpha(x)}\Phi$ leads the kinetic energy term to

$$\begin{aligned}&(\partial^\mu\Phi^* + ie_\Gamma\Gamma A^\mu\Phi^*)(\partial_\mu\Phi - ie_\Gamma\Gamma A_\mu\Phi)\\ &\quad + (e^{i\alpha}\partial^\mu e^{-i\alpha})\Phi^*(\partial_\mu\Phi - ie_\Gamma\Gamma A_\mu\Phi)\\ &\quad + (e^{-i\alpha}\partial_\mu e^{i\alpha})(\partial^\mu\Phi^* + ie_\Gamma\Gamma A^\mu\Phi^*)\Phi\\ &\quad + (\partial^\mu e^{-i\alpha})(\partial_\mu e^{i\alpha})\Phi^*\Phi.\end{aligned} \tag{5}$$

If we consider the global transformation $\mathrm{U}(1)_\Gamma$ below the anomalous scale, only the first term survives in the above equation,

$$\mathrm{U}(1)_\Gamma\ \Phi: \quad (\partial^\mu\Phi^* + ie_\Gamma\Gamma\tilde{Z}^\mu\Phi^*)(\partial_\mu\Phi - ie_\Gamma\Gamma\tilde{Z}_\mu\Phi) \tag{6}$$

where we expressed the $\mathrm{U}(1)_{\rm anom}$ gauge boson as $\tilde{Z}$. Below the anomalous scale, it describes a global symmetry $\mathrm{U}(1)_\Gamma$ coupling with the heavy anomalous gauge boson with the same charge Γ. In the potential V, this gauge boson coupling respects the global symmetry also. Thus, we obtain an exact global symmetry $\mathrm{U}(1)_\Gamma$ below the anomalous scale.

Thus, an intermediate scale global symmetry is from the anomalous $\mathrm{U}(1)_{\rm anom}$ in the compactification process of the heterotic string. Since the anomalous $\mathrm{U}(1)_{\rm anom}$ has the same coupling to gauge fields, we have the following MI axion coupling,

$$\begin{aligned}\mathcal{L} &= \frac{P}{f}\frac{g_2^2}{32\pi^2}W^a_{\mu\nu}\tilde{W}^{a\,\mu\nu} + \frac{P}{f}\frac{g_1^2}{32\pi^2}Y_{1,\mu\nu}\tilde{Y}_1^{\mu\nu}\\ &= \frac{P}{f}\frac{g_2^2}{32\pi^2}\left(W^a_{\mu\nu}\tilde{W}^{a\,\mu\nu} + (1/C^2)Y_{1,\mu\nu}\tilde{Y}_1^{\mu\nu}\right)\end{aligned}$$

Table 1
The axion–photon–photon coupling for several invisible axion models. The third row in superstring corresponds to the exact global symmetry $U(1)_{PQ} = U(1)_{anom}$ in the superpotential W. The MI axion with $f_a > 10^{16}$ GeV [28] has the same value as that of the $U(1)_{anom}$ string. The non-SUSY DFSZ models have a fine-tuning problem. One related cosmological problem even within SUSY framework was pointed out in [33]. In the KSVZ column, (m, m) means m numbers of $Q_{em} = +\frac{2}{3}$ quark and m numbers of $Q_{em} = -\frac{1}{3}$ quark.

	KSVZ	DFSZ					Superstring	Comments
Q_{em}	$c_{a\gamma\gamma}$	x	q^c-e_L pair	$c_{a\gamma\gamma}$	$c_{a\gamma\gamma}$		$c_{a\gamma\gamma}$	
0	-2	any x	(d^c, e)	$\frac{2}{3}$	$\frac{2}{3}$	arXiv:1405.6175	$\frac{2}{3}$	Anomalous U(1) as $U(1)_{PQ}$
$\pm\frac{1}{3}$	$-\frac{4}{3}$	any x	(u^c, e)	$-\frac{4}{3}$	$\frac{2}{3}$	hep-ph/0612107	$-\frac{1}{3}$	Approximate $U(1)_{PQ}$
$\pm\frac{2}{3}$	$\frac{2}{3}$			Without	GUTs or	This paper	$\geq\frac{2}{3}$	Anomalous U(1) as $U(1)_{PQ}$
± 1	4			SUSY	SUSY			$c_{a\gamma\gamma} = (1 - 2\sin^2\theta_W)/\sin^2\theta_W$
(m, m)	$-\frac{1}{3}$			H_d or H_u^*	H_d or H_u			with $m_u/m_d = 0.5$.

$$
\begin{aligned}
&= \frac{P}{f}\frac{g_2^2}{32\pi^2}\left(W^a_{\mu\nu}\tilde{W}^{a\,\mu\nu} + Y_{\mu\nu}\tilde{Y}^{\mu\nu}\right) \\
&\to \frac{P}{f}\frac{g_2^2}{32\pi^2}\left(2W^+_{\mu\nu}\tilde{W}^{-\,\mu\nu} + F^{\rm em}_{\mu\nu}\tilde{F}^{{\rm em}\,\mu\nu} + Z_{\mu\nu}\tilde{Z}^{\mu\nu}\right) \\
&= \frac{P}{f}\frac{1}{32\pi^2}\Bigg(2g_2^2 W^+_{\mu\nu}\tilde{W}^{-\,\mu\nu} + g_2^2 Z_{\mu\nu}\tilde{Z}^{\mu\nu} \\
&\quad + \frac{e^2}{\sin^2\theta_W}F^{\rm em}_{\mu\nu}\tilde{F}^{{\rm em}\,\mu\nu}\Bigg) \qquad (7)
\end{aligned}
$$

where Y_1 is the properly normalized U(1) gauge field. Note that $g' = Cg_1, s_W^2 = g'^2/G^2 = C^2 g_1^2/(C^2 g_1^2 + g_2^2) = 1/(1 + 1/C^2)$, and $c^0_{a\gamma\gamma} = \frac{1}{\sin^2\theta_W}$. Note that $C^2 = \frac{5}{3}$ in the SU(5) model. Here, we used

$$
\begin{aligned}
W^3_\mu &= \cos\theta_W Z_\mu + \sin\theta_W A_\mu, \\
Y_\mu &= -\sin\theta_W Z_\mu + \cos\theta_W A_\mu, \\
c_W &= \cos\theta_W = \frac{g_2}{\sqrt{g_2^2 + g'^2}}, \qquad (8) \\
s_W &= \sin\theta_W = \frac{g'}{\sqrt{g_2^2 + g'^2}}.
\end{aligned}
$$

Therefore, we obtain the following coupling for the anomalous case,

$$
c_{a\gamma\gamma} = \frac{1 - 2\sin^2\theta_W}{\sin^2\theta_W} \qquad (9)
$$

where we used $\frac{m_u}{m_d} = \frac{1}{2}$.

The axion–photon–photon couplings for invisible axions are summarized in Table 1. In the DFSZ model, the SM doublets H_u and H_d carry the PQ charges. So, their VEVs enter into the calculation and x is their ratio defined as $x = \tan\beta = v_u/v_d$. In the DFSZ columns, the case with H_u^* corresponds to that the $Q_{em} = -1$ leptons obtain mass by the coupling $f_{ij}(\tilde{H}_u^T \bar{e}^i_R \ell^j_L)$ where $\tilde{H}_u = i\sigma_2 H_u^*$, and $\ell^j_L = (\nu_j, e_j)^T_L$. In GUTs, both d^c or u^c has the same PQ charge as that of ℓ_i and $c_{a\gamma\gamma}$ are the same. With SUSY, holomorphy forbids the coupling of H_u^* to ℓ_i and only the coupling of H_d to ℓ_i is allowed.

2.3. The weak mixing angle in GUTs with extra U(1)'s

If the electromagnetic charge operator is embedded in a simple GUT group SU(N), the charge operator on the fundamental representation is a traceless matrix,

$$
\begin{aligned}
&Q_{\rm em}(\mathbf{N}) = {\rm diag.}(a, a, a, 0, -1, b_6, \cdots), \\
&3a - 1 + \sum_i b_i = 0. \qquad (10)
\end{aligned}
$$

In the Georgi–Glashow (GG) SU(5) model [36], $a = \frac{1}{3}$ and $b_i = 0$. If $Q_{\rm em}$ is completed by the simple SU(N) generators, the information on the fundamental representation is enough since the other higher dimensional representations can be constructed in terms of direct products of fundamentals. At the GUT scale three gauge couplings are the same, and the mixing angle is defined as $\sin\theta_W = e/g_2$. For properly normalized generators Q_1 and T_3 in the fundamental representation, the trace is $\frac{1}{2}$, and we have

$$
{\rm Tr}\,(eQ_{\rm em})^2 = {\rm Tr}\,(g_1 Q_1)^2 = {\rm Tr}\,(g_2 T_3)^2, \qquad (11)
$$

where $g_1 = g_2$ at the GUT scale. Thus, we obtain

$$
\sin^2\theta_W = \frac{e^2}{g_2^2} = \frac{{\rm Tr}\ (T_3)^2}{{\rm Tr}\,(Q_{\rm em})^2}. \qquad (12)
$$

For the SU(5) model, it is $(1/2)/(4/3) = 3/8$. For the electromagnetic charge (10) in SU(N), the mixing angle is

$$
\sin^2\theta_W = \frac{1/2}{3a^2 + 1 + \sum_i b_i^2}. \qquad (13)
$$

The SU(7) model of Ref. [37] gives $\sin^2\theta_W = 3/20$.

If electromagnetically neutral $SU(3)_c \times SU(2)_W \times U(1)_Y$ singlets are added to the fifteen chiral fields of SU(5), the weak mixing angle presented in Eq. (12) remains the same. We can present the following general statement. Suppose that a GUT group breaks at one scale $M_{\rm GUT}$ and matter fields break down to 45 chiral fields of the SM plus $SU(3)_c \times SU(2)_W \times U(1)_Y$ singlets,

$$
3\left\{q_L, u^c_L, d^c_L, \ell_L, \nu_L, e_L, e^c_L\right\} + \text{singlets}, \quad \text{at} \quad M_{\rm GUT}. \qquad (14)
$$

Then, Eq. (12) can be applied. Therefore, the SO(10) GUT has the weak mixing angle $\sin^2\theta_W = \frac{3}{8}$. It does not depend on how the symmetry breaking chain takes, through the GG SU(5) or through the flipped SU(5) [35,39], because there is only one scale $M_{\rm GUT}$. In the flipped-SU(5), there are three fermionic representations, $\mathbf{10}_{+1/5}, \overline{\mathbf{5}}_{-3/5}$, and $\mathbf{1}_{+1}$. If we consider all representations, Eq. (12) is still applicable.

However, if there are two scales for the symmetry breaking pattern such as SO(10) → flipped-SU(5) → SM, the weak mixing angle at the lower GUT scale has a logarithmic correction because $U(1)_{\rm em}$ is composed of two U(1)'s. For the electromagnetic charge operator composed of two U(1) couplings, *i.e.* e_N^2 from SU(N) part and $e^2_{(1')}$ from U(1)′ part, the electromagnetic charge is given by

$$
\frac{1}{e^2} = \frac{1}{e_N^2} + \frac{1}{e^2_{(1')}}. \qquad (15)
$$

If a vectorlike representation of the form $\mathbf{5}_{-a} + \overline{\mathbf{5}}_a$ is present in the model, then Eq. (12) is still applicable [32]. But, if $\mathbf{5}$ or $\overline{\mathbf{5}}$ does not appear in the anomaly-free combination as that from $\mathbf{16}$, that fundamental representation cannot be used in Eq. (12). The Higgs $\mathbf{5}_{-2/5}$ and $\overline{\mathbf{5}}_{2/5}$ in the flipped-SU(5) give $\sin^2\theta_W = \frac{3}{8}$ via Eq. (15).

The definition of c_3 and $c^0_{a\gamma\gamma}$ given in Eq. (1) dictates that f_a is the vacuum expectation value $\langle\sigma\rangle$ divided by the domain wall

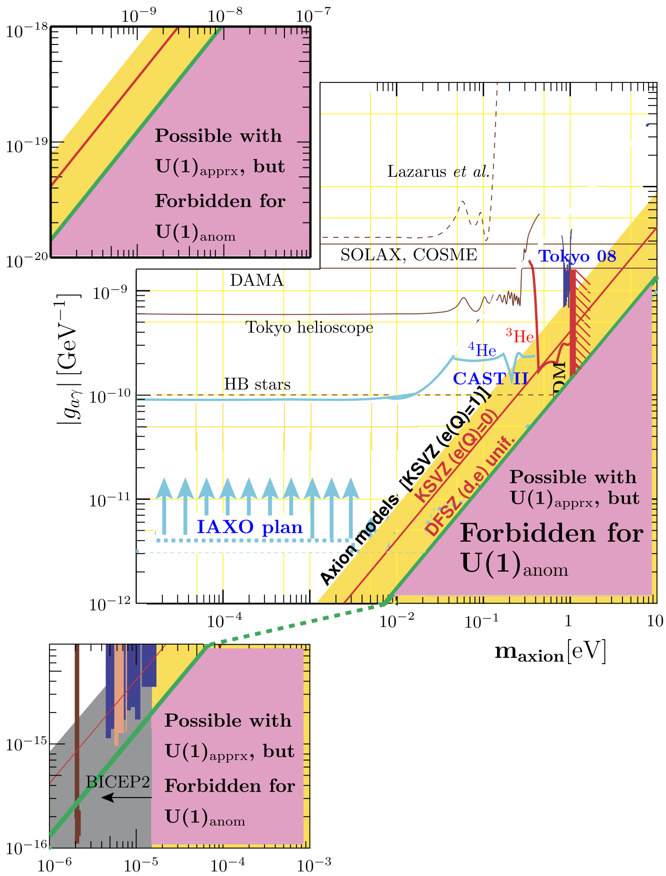

Fig. 2. The allowed parameter space of $g_{a\gamma}\,[\mathrm{GeV}^{-1}] = 1.57 \cdot 10^{-10}\, c_{a\gamma\gamma}$ vs. axion mass. The RHS of this is dimensionless as written, but it is interpreted as $[\mathrm{GeV}^{-1}]$, as marked in the vertical axis and in the LHS of this equation. The lavender part is not allowed if the $U(1)_{PQ}$ is the anomalous $U(1)_{anom}$. However, it can be allowed for some approximate $U(1)_{PQ}$, as shown in Ref. [34] for the flipped-SU(5) model of Ref. [35]. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

number N_{DW}. $c^0_{a\gamma\gamma}$ is defined relative to c_3 with c_3 taking into account N_{DW}. For three fundamental representations, c_3 is three times that of one fundamental, and also $c^0_{a\gamma\gamma}$ is three times that of one fundamental. The color coupling defines f_a as $\langle\sigma\rangle/N_{DW}$. Thus, $c^0_{a\gamma\gamma}$ is defined relative to c_3, *i.e.* $c^0_{a\gamma\gamma} = \mathrm{Tr}\,(Q_{em})^2/\mathrm{Tr}\,(F_3)^2$ where F_3 is one generator of color gauge group $SU(3)_c$. For a fundamental representation, $\mathrm{Tr}\,(F_3)^2 = \mathrm{Tr}\,(T_3)^2$ where T_3 is one generator of weak gauge group $SU(2)_W$, and we obtain

$$c^0_{a\gamma\gamma} = \frac{\mathrm{Tr}\,(Q_{em})^2}{\mathrm{Tr}\,(T_3)^2} = \frac{1}{\sin^2\theta_W}, \tag{16}$$

if one fundamental representation is enough to calculate $c^0_{a\gamma\gamma}$. Because the SM, represented in Eq. (14), has the contribution from three families

$$\frac{\mathrm{Tr}\,(Q_{em})^2}{\mathrm{Tr}\ (F_3)^2} = \frac{8}{3}, \tag{17}$$

the extra charged singlets will make this contribution larger. Therefore, GUTs predict

$$c^0_{a\gamma\gamma} \geq \frac{8}{3}. \tag{18}$$

Thus, we have the excluded region for the case of anomalous U(1) being $U(1)_{PQ}$ in Fig. 2. However, if $U(1)_{PQ}$ is approximate as calculated in a string compactification [34], this bound does not apply.

3. A simple calculation of $c^0_{a\gamma\gamma}$ from quantum numbers

In this section, we show a simple method for calculating the entries in the DFSZ models in Table 1. Let the invisible axion is housed in the complex singlet σ [1]. The DFSZ model connects the PQ charges of H_u and H_d to that of σ. One possible connection is

$$H_u H_d \sigma^2, \tag{19}$$

and the PQ charge Γ of σ is assigned to be +1. The mass terms of the up- and down-type quarks are

$$H_u^\dagger \bar{u}_R q_L,\ H_d^\dagger \bar{d}_R q_L, \tag{20}$$

where q_L and ℓ_L are $SU(2)_W$ doublets.

If the charged leptons obtain mass by H_d via $H_d^\dagger \bar{e}_R \ell_L$, we can assign the charges of H_u, q_L, ℓ_L, and u_R zero. Then, H_d carries −2 units of the charge. The charges of d_R and e_R are +2. Certainly, this definition is free of gauge charges since fermions d_R and e_R, having different SM gauge charges, have the same charge. Namely, these charges can be defined not to contain gauge charges. Even if $U(1)_{PQ}$ contains a component aligned with $U(1)_{em}$, the result does not change since the gauged $U(1)_{em}$ does not have any gauge anomalies. So, −2 can be considered wholly as the global PQ charge. Thus, $\Gamma - Q_{em} - Q_{em}$ anomaly is proportional to $+2e^2[3(-1/3)^2 + (-1)^2] = \frac{8e^2}{3}$, which is used in the table for (d^c, e) unification.

If the charged leptons obtain mass by H_u via $\tilde{H}_u^\dagger \bar{e}_R \ell_L$, we can assign the PQ charges of H_d, q_L, ℓ_L, and d_R zero. Since H_u carries −2 units of the PQ charge, the PQ charge of u_R is +2, and the PQ charge of e_R is −2. Thus, $\Gamma - Q_{em} - Q_{em}$ anomaly is proportional to $+2e^2[3(+2/3)^2 - (-1)^2] = \frac{2e^2}{3}$, which is used in the table for (u^c, e) unification. In SUSY models, H_u cannot be used for the electron mass due to the holomorphic condition and the weak hypercharge.

4. Conclusion

For the exact global symmetry $U(1)_{PQ}$ from string compactification, we obtained the lower bound, $\frac{8}{3} - c^{\mathrm{ch\,br}}_{a\gamma\gamma}$, for the axion–photon–photon coupling $c_{a\gamma\gamma}$, where $c^{\mathrm{ch\,br}}_{a\gamma\gamma} \simeq 2$, and presented it in Table 1 and Fig. 2 together with other models. This bound is free from the gravity obstruction of global symmetries. However, if $U(1)_{PQ}$ is approximate, this bound does not apply. We based our argument from the orbifold compactification, which has some limitation as stated in Sec. 2. Finally, let us present a caveat. In the effective field theory of orbifold models, due to the Fayet–Iliopoulos term associated to the anomalous U(1) symmetry, one has to give VEVs to some chiral multiplets in order to cancel the Fayet–Iliopoulos piece. On the geometrical side, this can be understood as an instability of the orbifold point and the fields having VEVs as resolution modes smoothening the orbifold singularities. Whether the $U(1)_{PQ}$ is broken or not depends on the number of required VEVed fields to smooth out the orbifold singularity. If there were one gauged U(1) and one global U(1), and only one VEVed field is needed, then a global symmetry remains unbroken by the so-called 't Hooft mechanism [40]. Thus, if the needed number of independent VEVed fields, to smooth out the orbifold singularities, is the same or less than the number of unbroken U(1) gauge groups, then the $U(1)_{PQ}$ survives down to the low energy scale.

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