

Natural inflation with multiple sub-Planckian axionsKiwoon Choi,^{1,*} Hyungjin Kim,^{1,2,†} and Seokhoon Yun^{1,2,‡}¹*Center for Theoretical Physics of the Universe, Institute for Basic Science (IBS), Daejeon 305-811, Korea*²*Department of Physics, KAIST, Daejeon 305-701, Korea*

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We extend the Kim-Nilles-Peloso (KNP) alignment mechanism for natural inflation to models with $N > 2$ axions, which obtains super-Planckian effective axion decay constant $f_{\text{eff}} \gg M_{\text{Pl}}$ through an alignment of the anomaly coefficients of multiple axions having sub-Planckian fundamental decay constants $f_0 \ll M_{\text{Pl}}$. The original version of the KNP mechanism realized with two axions requires that some of the anomaly coefficients should be of the order of f_{eff}/f_0 , which would be uncomfortably large if $f_{\text{eff}}/f_0 \gtrsim \mathcal{O}(100)$ as suggested by the recent BICEP2 results. We note that the KNP mechanism can be realized with the anomaly coefficients of $\mathcal{O}(1)$ if the number of axions N is large as $N \ln N \gtrsim 2 \ln(f_{\text{eff}}/f_0)$, in which case the effective decay constant can be enhanced as $f_{\text{eff}}/f_0 \sim \sqrt{N!} n^{N-1}$ for n denoting the typical size of the integer-valued anomaly coefficients. Comparing to the other multiple axion scenario, the N -flation scenario which requires $N \sim f_{\text{eff}}^2/f_0^2$, the KNP mechanism has a virtue of not invoking to a too large number of axions, although it requires a specific alignment of the anomaly coefficients, which can be achieved with a probability of $\mathcal{O}(f_0/f_{\text{eff}})$ under a random choice of the anomaly coefficients. We also present a simple model realizing a multiple axion monodromy along the inflaton direction.

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I. INTRODUCTION

Recent detection of tensor modes in the cosmic microwave background by BICEP2 suggests that the inflation scale is rather high, $H_I \sim 10^{14}$ GeV [1]. In the context of slow roll inflation, such a high inflation scale implies that the inflaton field has experienced a super-Planckian excursion over the inflation period [2]. This poses a question if the inflaton field can be decoupled from high scale physics above the scale of quantum gravity, so that an effective field theory description of inflation does make sense.

An attractive solution to this puzzle is to introduce an approximate continuous shift symmetry along the inflaton direction. To implement this idea, in natural inflation [3] the inflaton is assumed to be a pseudo-Nambu-Goldstone boson ϕ with a periodic potential

$$V(\phi) = \Lambda^4 \left[1 - \cos\left(\frac{\phi}{f}\right) \right], \quad (1)$$

where f is the axion decay constant which defines the fundamental domain of the axionlike inflaton field:

$$\phi \equiv \phi + 2\pi f. \quad (2)$$

This inflaton potential is stable against perturbative quantum corrections, which can be assured through the

approximate shift symmetry $\phi \rightarrow \phi + \text{constant}$. Also, under a reasonable assumption on the nonperturbative dynamics generating the axion potential, one can justify that the above simple potential provides a good approximation to the full inflaton potential.

However, there is a difficulty in this simple setup. For successful slow roll inflation, the axion decay constant is required to have a super-Planckian value. The recent BICEP2 result makes the problem even more severe as it suggests [4]

$$f \gtrsim 10M_{\text{Pl}}, \quad (3)$$

where $M_{\text{Pl}} \simeq 2.4 \times 10^{18}$ GeV is the reduced Planck mass. It appears to be difficult to get such a large axion decay constant from a sensible fundamental theory, particularly from string theory. In the limit where a controllable approximation is available, string theory predicts that the axion scale is typically around $g^2 M_{\text{Pl}}/8\pi^2$ [5].

During the past years, there have been several proposals to circumvent this problem. They include the two-axion model of Kim *et al.* [6], which obtains a super-Planckian effective axion decay constant through an alignment of the anomaly coefficients of two axions having sub-Planckian fundamental decay constants, a five-dimensional (5D) gauge-axion unification model in which the axionlike inflaton obtains a super-Planckian decay constant in the limit that 5D gauge coupling becomes weaker than the 5D gravitational coupling [7], the N -flation scenario [8] based on the idea of assisted inflation [9] with many axions, and the axion monodromy based on either a string theoretic

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[10,11] or field theoretic [12,13] scheme for multiple windings in the axion field space. In this paper, we revisit the Kim-Nilles-Peloso (KNP) alignment mechanism to extend the scheme to models with $N > 2$ axions.

The original version of the KNP mechanism realized with two axions requires that some of the anomaly coefficients should be of the order of f_{eff}/f_i , where $f_{\text{eff}} \gg M_{\text{Pl}}$ is the super-Planckian effective decay constant of the axionic inflaton, while $f_i \ll M_{\text{Pl}}$ are the sub-Planckian fundamental axion decay constants in the model. In case that $f_i \sim g^2 M_{\text{Pl}}/8\pi^2$ [5] as suggested by string theory, this would require that some anomaly coefficients should be uncomfortably large as $f_{\text{eff}}/f_i = \mathcal{O}(10^2-10^3)$. We note that the KNP mechanism can be realized with the anomaly coefficients of $\mathcal{O}(1)$ if the number of axions is large as $N \ln N \gtrsim 2 \ln(f_{\text{eff}}/f_i)$, in which case the effective decay constant can be enhanced as $f_{\text{eff}}/f_i \sim \sqrt{N!} n^{N-1}$ for n denoting the typical size of the integer-valued anomaly coefficients. We examine also the probability for the KNP alignment to be achieved under a random choice of the anomaly coefficients.

In regard to enhancing the effective axion decay constant, a relevant question is how many fields do we need to get super-Planckian $f_{\text{eff}} \gg M_{\text{Pl}}$. As the Planck scale receives a quadratically divergent radiative correction from each light field, schematically we have $\delta M_{\text{Pl}}^2 \propto N_l \Lambda^2$, where N_l denotes the number of light fields and Λ is the cutoff scale of loop momenta. Then the scheme would be in trouble if it requires a too large number of light fields as $N_l \geq f_{\text{eff}}^2/f_i^2$. In our multiple axion scenario, f_{eff}/f_i grows exponentially as a function of N for a fixed value of $n > 1$, so the number of required axions is of the order of $\ln(f_{\text{eff}}/f_i)$. On the other hand, for the original two-axion KNP model [6], one needs $n = \mathcal{O}(f_{\text{eff}}/f_i)$, where the anomaly coefficient n can be identified as the number of gauge-charged fermions generating the axion coupling to instantons. In the N -flation scenario [8], the number of required axions is $\mathcal{O}(f_{\text{eff}}^2/f_i^2)$. So our scheme can enhance f_{eff} by introducing a parametrically smaller number of fields, as compared to the KNP two-axion model and the N -flation scenario.

In certain cases, the KNP mechanism can be interpreted as enhancing the effective axion decay constant as $f_{\text{eff}}/f_i \sim n \gg 1$ through the Z_n monodromy structure of a light axion, which is induced along the inflaton direction by the mixing with heavy axions. In this context, we present a simple model yielding $f_{\text{eff}}/f_i \sim \prod_{i=2}^N n_i$ through a multiple axion monodromy described by $\prod_{i=2}^N Z_{n_i}$ ($n_i > 1$). We present also a model yielding $f_{\text{eff}}/f_i \sim 2^{N-1}$ even when all the integer-valued anomaly coefficients are restricted as $|n_{ij}| \leq 1$.

The organization of this paper is as follows. In Sec. II, we review the original two-axion model of KNP to illustrate the basic idea and set the notations. In Sec. III, we extend the KNP mechanism to models with $N > 2$ axions. Sec. IV is the conclusion.

II. KIM-NILLES-PELOSO MECHANISM WITH TWO AXIONS

We begin with a brief review of the original Kim-Nilles-Peloso mechanism realized with two axions [6]. In the field basis of *periodic* axions:

$$\phi_i \equiv \phi_i + 2\pi f_i, \quad (4)$$

the axion potential consistent with the axion periodicity is generically given by

$$V(\phi_i) = \Lambda_1^4 \left[1 - \cos \left(\frac{n_1 \phi_1}{f_1} + \frac{n_2 \phi_2}{f_2} \right) \right] + \Lambda_2^4 \left[1 - \cos \left(\frac{m_1 \phi_1}{f_1} + \frac{m_2 \phi_2}{f_2} \right) \right], \quad (5)$$

where $\vec{n} = (n_1, n_2)$ and $\vec{m} = (m_1, m_2)$ are linearly independent integer-valued coefficients, and f_i ($i = 1, 2$) denote the fundamental axion decay constants which are presumed to be comparable to each other, while being significantly lower than the reduced Planck scale:

$$f_1 \sim f_2 \ll M_{\text{Pl}}.$$

Here we include only the leading nonperturbative effects generating the axion potential, under the assumption that the next order nonperturbative effects are small enough.

The integer-valued coefficients n_i, m_i parametrize the discrete degrees of freedom in the underlying nonperturbative dynamics generating the axion potential. A simple possibility is that the axion potential is generated by hidden sector gauge field instantons through the symmetry breaking by anomalies. In such case, the model involves two non-Abelian hidden sector gauge groups G_a ($a = 1, 2$), together with the gauge-charged fermions having the following couplings to axions:

$$\sum_I \sum_i \lambda_{iI} f_i e^{iq_{iI} \phi_i / f_i} \bar{\psi}_{iL} \psi_{iR} + \text{H.c.}, \quad (6)$$

where λ_{iI} denote dimensionless Yukawa couplings and ψ_I are assumed to be charged Dirac fermions for simplicity. Then the Noether current of the nonlinearly realized Peccei-Quinn symmetries

$$U(1)_i: \phi_i \rightarrow \phi_i + \alpha_i f_i, \quad \bar{\psi}_{iL} \psi_{iR} \rightarrow e^{-iq_{iI} \alpha_i} \bar{\psi}_{iL} \psi_{iR} \quad (7)$$

have the $U(1)_i$ - G_a - G_a anomalies as

$$\partial_\mu J_i^\mu = \frac{n_i}{16\pi^2} F_1 \tilde{F}_1 + \frac{m_i}{16\pi^2} F_2 \tilde{F}_2, \quad (8)$$

where F_a are the gauge field strength of the gauge group G_a , and the anomaly coefficients are given by

$$n_i = 2 \sum_I q_{iI} \text{Tr}(T_1^2(\psi_I)), \quad m_i = 2 \sum_I q_{iI} \text{Tr}(T_2^2(\psi_I)) \quad (9)$$

for $T_a(\psi_I)$ ($a = 1, 2$) denoting the G_a -charge matrix of ψ_I normalized as $\text{Tr}(T_a^2) = 1/2$ for the fundamental representation of G_a . With this symmetry breaking by anomalies, the gauge field instantons of G_a generate the axion potential of the form (5). Based on this observation, in the following we will call n_i, m_i the anomaly coefficients. However it should be noted that the axion potential (5) can be generated by different kinds of nonperturbative effects, for instance string theoretic instantons or hidden gaugino condensations. In such case, the integer coefficients n_i, m_i can be determined by a variety of different discrete UV quantum numbers, e.g. the quantized fluxes, the number of stacked D -branes, and/or the number of windings for stringy instantons.

To discuss the KNP mechanism, it is convenient to consider the limit $\Lambda_2 = 0$, in which the axion potential is given by

$$V(\phi_i) = \Lambda_1^4 \left[1 - \cos \left(\frac{n_1 \phi_1}{f_1} + \frac{n_2 \phi_2}{f_2} \right) \right]. \quad (10)$$

Obviously this axion potential has a one-dimensional periodic flat direction in the two-dimensional fundamental domain of the axion fields spanned by $\phi_i = [0, 2\pi f_i]$,

$$\phi_{\text{flat}} \propto \frac{n_2 \phi_1}{f_2} - \frac{n_1 \phi_2}{f_1}, \quad (11)$$

which can be identified as the inflaton direction. One easily finds that the length of this periodic flat direction is given by

$$\ell_{\text{flat}} = \frac{2\pi \sqrt{n_1^2 f_2^2 + n_2^2 f_1^2}}{\text{gcd}(n_1, n_2)}, \quad (12)$$

where $\text{gcd}(n_1, n_2)$ denotes the greatest common divisor of n_1 and n_2 . This shows that a super-Planckian flat direction with $\ell_{\text{flat}} \gg M_{\text{Pl}} \gg f_i$ can be developed on the two-dimensional sub-Planckian domain if

$$\frac{n_1}{\text{gcd}(n_1, n_2)} \quad \text{or} \quad \frac{n_2}{\text{gcd}(n_1, n_2)} \gg \frac{M_{\text{Pl}}}{f_i} \gg 1. \quad (13)$$

In Fig. 1, we depict the flat direction in the fundamental domain of axion fields, which has a length given by (12). Since the axionic inflaton of natural inflation rolls down along this periodic flat direction, its effective decay constant is bounded as

$$f_{\text{eff}} \leq \frac{\ell_{\text{flat}}}{2\pi},$$

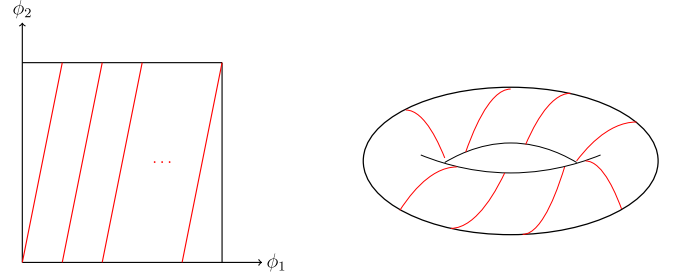


FIG. 1 (color online). Flat direction in the fundamental domain of axion fields in the limit $\Lambda_2 = 0$. Even though the fundamental domain is sub-Planckian with $f_i \ll M_{\text{Pl}}$, the flat direction can have a super-Planckian length if one (or both) of $n_i/\text{gcd}(n_1, n_2)$ is large enough. The right panel depicts the flat direction in the fundamental domain for which the axion periodicity is manifest.

which means that at least one of n_i should be as large as $\text{gcd}(n_1, n_2) f_{\text{eff}}/f_i$.

Turning on the second axion potential

$$\Delta V = \Lambda_2^4 \left[1 - \cos \left(\frac{m_1 \phi_1}{f_1} + \frac{m_2 \phi_2}{f_2} \right) \right], \quad (14)$$

a nontrivial potential is developed along the periodic flat direction having a length (12). Even when $\ell_{\text{flat}} \gg M_{\text{Pl}}$, natural inflation is not guaranteed as the inflaton potential induced by ΔV generically has multiple modulations along the flat direction. We find that the number of modulations over the full range of the periodic flat direction is given by

$$N_{\text{mod}} = \frac{|n_1 m_2 - n_2 m_1|}{\text{gcd}(n_1, n_2)}, \quad (15)$$

which results in the effective decay constant

$$f_{\text{eff}} = \frac{\ell_{\text{flat}}}{2\pi N_{\text{mod}}} = \frac{\sqrt{n_1^2 f_2^2 + n_2^2 f_1^2}}{|n_1 m_2 - n_2 m_1|}. \quad (16)$$

It is straightforward to derive the above form of the effective axion decay constant [6]. Taking the rotated axion field basis:

$$\psi = \frac{n_1 f_2 \phi_1 + n_2 f_1 \phi_2}{\sqrt{n_1^2 f_2^2 + n_2^2 f_1^2}}, \quad \phi_{\text{flat}} = \frac{n_2 f_1 \phi_1 - n_1 f_2 \phi_2}{\sqrt{n_1^2 f_2^2 + n_2^2 f_1^2}}, \quad (17)$$

the axion potential (5) can be written as

$$V(\psi, \phi_{\text{flat}}) = \Lambda_1^4 \left[1 - \cos \left(\frac{\psi}{f_\psi} \right) \right] + \Lambda_2^4 \left[1 - \cos \left(\frac{\psi}{f'_\psi} + \frac{\phi_{\text{flat}}}{f_{\text{eff}}} \right) \right], \quad (18)$$

where

$$\begin{aligned}
f_\psi &= \frac{f_1 f_2}{\sqrt{n_1^2 f_2^2 + n_2^2 f_1^2}}, \\
f'_\psi &= \frac{f_1 f_2 \sqrt{n_1^2 f_2^2 + n_2^2 f_1^2}}{n_1 m_1 f_2^2 + n_2 m_2 f_1^2}, \\
f_{\text{eff}} &= \frac{\sqrt{n_1^2 f_2^2 + n_2^2 f_1^2}}{|n_1 m_2 - n_2 m_1|}.
\end{aligned} \tag{19}$$

Taking the limit $\Lambda_1^4 \gg \Lambda_2^4$, the heavy field component ψ can be integrated out, yielding the effective potential of the light inflaton direction ϕ_{flat} as

$$V_{\text{eff}}(\phi_{\text{flat}}) = \Lambda_2^4 \left[1 - \cos\left(\frac{\phi_{\text{flat}}}{f_{\text{eff}}}\right) \right]. \tag{20}$$

From the above expression of f_{eff} , it is clear that $f_{\text{eff}} \gg f_i$ is *not* a generic feature of the model, but requires a special alignment of the anomaly coefficients n_i, m_i . Note that in the absence of any alignment, generically

$$f_{\text{eff}} \sim \frac{f_i}{n}, \tag{21}$$

where n represents the typical size of the anomaly coefficients n_i, m_i . Obviously there can be many different choices (or alignments) of the anomaly coefficients yielding $f_{\text{eff}}/f_i \gg 1$. A simple possibility is that one or both of n_i are large, while the denominator $(n_1 m_2 - n_2 m_1)$ is tuned to be $\mathcal{O}(1)$. As a specific example, KNP considered the case that $n_2 \gg 1$ and the other three anomaly coefficients are given by $m_1 = n_1 = 1$ and $m_2 = n_2 + \mathcal{O}(1)$ [6]. Of course, one can consider different examples as we do below, for instance $n_2 \gg 1$ with $n_1 = m_2 = 1, m_1 = 0$.

To understand the geometric meaning of the required alignment, one can introduce an alignment angle $\delta\theta$ as

$$\sin \delta\theta \equiv \frac{|n_1 m_2 - n_2 m_1|}{\sqrt{(n_1^2 + n_2^2)(m_1^2 + m_2^2)}}, \tag{22}$$

and rewrite the effective decay constant as

$$f_{\text{eff}} = \frac{1}{\sin \delta\theta} \left(\frac{n_1^2 f_2^2 + n_2^2 f_1^2}{(n_1^2 + n_2^2)(m_1^2 + m_2^2)} \right)^{1/2}. \tag{23}$$

Note that $\delta\theta$ corresponds to the angle between the heavy field direction ψ and the other field direction ϕ of the second axion potential ΔV :

$$\psi \propto \frac{n_1 \phi_1}{f_1} + \frac{n_2 \phi_2}{f_2}, \quad \phi \propto \frac{m_1 \phi_1}{f_1} + \frac{m_2 \phi_2}{f_2}. \tag{24}$$

For the case of two axions, one of the economic ways to get $f_{\text{eff}}/f_i \gg 1$ is to have

$$n_1 \sim n_2 = \mathcal{O}(f_{\text{eff}}/f_i), \quad m_1 \sim m_2 = \mathcal{O}(1), \tag{25}$$

for which

$$\delta\theta = \mathcal{O}(f_i/f_{\text{eff}}). \tag{26}$$

This implies that the probability for achieving $f_{\text{eff}} \gg f_i$ under a random choice of the anomaly coefficients, but within the specific region of (25), is given by

$$P(f_{\text{eff}}/f_i) = \mathcal{O}(f_i/f_{\text{eff}}). \tag{27}$$

On the other hand, if one extends the random choice to the generic region where all the anomaly coefficients can be of the order of f_{eff}/f_i , one needs an alignment angle $\delta\theta = \mathcal{O}(f_i^2/f_{\text{eff}}^2)$, and then the probability for achieving $f_{\text{eff}} \gg f_i$ is reduced as

$$P(f_{\text{eff}}/f_i) = \mathcal{O}(f_i^2/f_{\text{eff}}^2). \tag{28}$$

A particularly interesting choice [14,15] of the anomaly coefficients realizing the KNP alignment is

$$n_1 = m_2 = 1, \quad m_1 = 0, \quad |n_2| \gg 1, \tag{29}$$

for which the light inflaton direction is identified as

$$\phi_{\text{flat}} = \frac{n_2 f_1 \phi_1 - f_2 \phi_2}{f_{\text{eff}}} \simeq \phi_1 \tag{30}$$

with the effective decay constant

$$f_{\text{eff}} = \sqrt{n_2^2 f_1^2 + f_2^2} \simeq |n_2| f_1 \tag{31}$$

where we assumed $f_1 \sim f_2$. In this case, we have

$$e^{i\phi_1/f_1} = e^{-in_2 \phi_2/f_2} \tag{32}$$

along the inflaton direction. Then the enhanced effective axion decay constant can be considered as a consequence of the Z_{n_2} monodromy structure of ϕ_1 , which is induced by the mixing with the heavy axion component ϕ_2 .

III. GENERALIZATION TO $N > 2$ AXIONS

It is in fact straightforward to generalize the KNP mechanism to the case with $N > 2$ axions.¹ In the presence of N axions, again in the periodic axion field basis

$$\phi_i \equiv \phi_i + 2\pi f_i \quad (i = 1, 2, \dots, N), \tag{33}$$

the axion potential takes the form

¹A generalization to the case with $N > 2$ axions has been discussed in [16] to accommodate the intermediate scale QCD axion as well as a quintessence axion with Planck scale decay constant in the model.

$$V = \sum_{i=1}^N \Lambda_i^4 \left[1 - \cos \left(\sum_{j=1}^N \frac{n_{ij} \phi_j}{f_j} \right) \right], \quad (34)$$

where $\vec{n}_i = (n_{i1}, n_{i2}, \dots, n_{iN})$ are linearly independent integer-valued anomaly coefficients, and $f_i \ll M_{\text{Pl}}$ are the fundamental decay constants which are presumed to be comparable to each other. To proceed, let us take the limit

$$\Lambda_N = 0,$$

for which the one-dimensional periodic flat direction is parametrized as

$$C_i = (-1)^{i+1} \text{Det} \begin{pmatrix} n_{11} & n_{12} & \dots & n_{1,(i-1)} & n_{1,(i+1)} & \dots & n_{1N} \\ n_{21} & n_{22} & \dots & n_{2,(i-1)} & n_{2,(i+1)} & \dots & n_{2N} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ n_{N-1,1} & n_{N-1,2} & \dots & n_{N-1,(i-1)} & n_{N-1,(i+1)} & \dots & n_{N-1,N} \end{pmatrix}. \quad (37)$$

The length of this periodic flat direction is determined by the minimal discrete shift $\Delta\phi_i$ along the flat direction, under which the axion field configuration comes back to the original configuration. One then finds

$$\Delta\phi_i = \frac{2\pi C_i f_i}{\text{gcd}(C_1, C_2, \dots, C_N)}, \quad (38)$$

yielding the length of the flat direction:

$$\ell_{\text{flat}} = \frac{2\pi \sqrt{\sum_{i=1}^N C_i^2 f_i^2}}{\text{gcd}(C_1, C_2, \dots, C_N)}, \quad (39)$$

where $\text{gcd}(C_1, C_2, \dots, C_N)$ denotes the greatest common divisor of all C_i .

For generic anomaly coefficients n_{ij} , the magnitude of C_i is quite sensitive to the number of axions, as well as to the typical size of n_{ij} . Here we are interested in the limit $N \gg 1$ with

$$|n_{ij}| \leq n_{\text{max}} = \mathcal{O}(\text{few}). \quad (40)$$

To proceed, we can regard n_{ij} as a random integer-valued variable with flat probability distribution:

$$P(n_{ij}) = \frac{1}{2n_{\text{max}} + 1}.$$

We then have

$$\begin{aligned} \langle n_{ij} \rangle &= \sum_{n_{ij}=-n_{\text{max}}}^{n_{\text{max}}} n_{ij} P(n_{ij}) = 0, \\ \langle n_{ij}^2 \rangle &= \sum_{n_{ij}=-n_{\text{max}}}^{n_{\text{max}}} n_{ij}^2 P(n_{ij}) = \frac{1}{3} n_{\text{max}} (1 + n_{\text{max}}) \equiv n^2, \end{aligned}$$

$$\phi_{\text{flat}} \propto \sum_i X_i \phi_i \equiv \text{Det} \begin{pmatrix} \phi_1 & \phi_2 & \dots & \phi_N \\ \frac{n_{11}}{f_1} & \frac{n_{12}}{f_2} & \dots & \frac{n_{1N}}{f_N} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{n_{N-1,1}}{f_1} & \frac{n_{N-1,2}}{f_2} & \dots & \frac{n_{N-1,N}}{f_N} \end{pmatrix}, \quad (35)$$

where

$$X_i = \frac{C_i f_i}{\prod_j f_j} \quad (36)$$

for

where

$$n = \sqrt{n_{\text{max}}(1 + n_{\text{max}})/3}$$

denotes the typical size of the anomaly coefficients in the range (40). One can similarly compute the expectation value of C_i^2 . For $C_N = \sum_{\sigma} \text{sgn}(\sigma) n_{1\sigma(1)} \dots n_{(N-1)\sigma(N-1)}$, where the summation is over all possible permutations of $\{1, 2, \dots, N-1\}$, one easily finds

$$\begin{aligned} \langle C_N^2 \rangle &= \left\langle \sum_{\sigma} (n_{1\sigma(1)} \dots n_{(N-1)\sigma(N-1)})^2 \right\rangle \\ &= (N-1)! n^{2(N-1)}, \end{aligned}$$

and therefore

$$\left\langle \sum_{i=1}^N C_i^2 \right\rangle = N! \cdot n^{2(N-1)}.$$

As implied by this expectation value, in most cases we have²

$$\left(\sum_{i=1}^N C_i^2 \right)^{1/2} \sim \sqrt{N!} n^{N-1}, \quad \text{gcd}(C_1, C_2, \dots, C_N) = \mathcal{O}(1), \quad (41)$$

²We have in fact an upper bound $\sqrt{\sum_i C_i^2} < N^{N/2} n_{\text{max}}^{N-1}$. We found through a numerical analysis that $\sqrt{\sum_i C_i^2} \gtrsim 0.2 \sqrt{N!} n^{N-1}$ in most cases of our interest. It is also known that a probability for $\text{gcd}(C_i) = 1$ under a random choice of C_i within the range $|C_i| \leq Q$ is given by $P(\text{gcd}(C_i) = 1) = 1/\zeta(N) + \mathcal{O}(1/Q)$, which is close to the unity in the limit $N \gg 1$ and $Q \gg 1$. Although in our case C_i is not a randomly chosen integer, but a determinant of the randomly chosen anomaly coefficients n_{ij} , we confirmed again through a numerical analysis that $\text{gcd}(C_i) = 1$ in most cases of our interest.

and therefore a flat direction enhanced as

$$\frac{\ell_{\text{flat}}}{2\pi f_i} \sim \sqrt{N!} n^{N-1}, \quad (42)$$

where the sub-Planckian axion decays constants f_i are assumed to be comparable to each other. Here we are interested in the case that the sub-Planckian axions ϕ_i originate from higher-dimensional tensor gauge fields in compactified string theory, in which case all f_i are comparable to $M_{st}/8\pi^2$ for the string scale M_{st} [5]. Note that the flat direction length is exponentially enhanced in the limit $N \gg 1$ when the typical anomaly coefficient $n > 1$. As a result, an exponentially long flat direction can be obtained with the anomaly coefficients $|n_{ij}| \leq n_{\text{max}} = \mathcal{O}(\text{few})$, with a moderately large number of axions

$$N \ln N \sim 2 \ln(\ell_{\text{flat}}/2\pi f_i). \quad (43)$$

This can be understood by noting that the flat direction generically winds each of the additional axion dimensions by about n times, which would explain the factor n^{N-1} , and there are also combinatoric degrees of freedom for the windings of the N -dimensional torus of axion fields, which would explain the factor $\sqrt{N!}$.

Introducing the last part of the axion potential

$$\Delta V = \Lambda_N^4 \left[1 - \cos \left(\sum_j \frac{n_{Nj} \phi_j}{f_j} \right) \right],$$

a periodic potential is developed along the flat direction. Again super-Planckian flat direction does not guarantee a super-Planckian effective decay constant. Instead we have

$$f_{\text{eff}} = \frac{\ell_{\text{flat}}}{2\pi N_{\text{mod}}}, \quad (44)$$

where N_{mod} is the number of modulations of the axion potential over the full period of the flat direction. Taking the limit $\Lambda_i \gg \Lambda_N$ ($i = 1, 2, \dots, N-1$) to integrate out the $(N-1)$ heavy axions, we find that the effective potential of the flat direction is given by

$$V_{\text{eff}}(\phi_{\text{flat}}) = \Lambda_N^4 \left[1 - \cos \left(\frac{\phi_{\text{flat}}}{f_{\text{eff}}} \right) \right], \quad (45)$$

where

$$f_{\text{eff}} = \frac{\sqrt{\sum_{i=1}^N C_i^2 f_i^2}}{|\text{Det}\mathbb{N}|}, \quad (46)$$

for

$$\begin{aligned} \text{Det}\mathbb{N} &= \text{Det} \begin{pmatrix} n_{N1} & n_{N2} & \cdots & n_{NN} \\ n_{11} & n_{12} & \cdots & n_{1N} \\ \vdots & \vdots & & \vdots \\ n_{(N-1)1} & n_{(N-1)2} & \cdots & n_{(N-1)N} \end{pmatrix} \\ &= \sum_i C_i n_{Ni}. \end{aligned} \quad (47)$$

This tells that the number of modulations over the full range of the flat direction is

$$N_{\text{mod}} = \frac{|\sum_i C_i n_{Ni}|}{\text{gcd}(C_1, C_2, \dots, C_N)}.$$

To justify our approach to integrate out the $(N-1)$ heavy axions, let us briefly examine the axion masses in our scheme. For the axion potential (34), the $N \times N$ axion mass-square matrix is given by $m_{kl}^2 = \sum_i \Lambda_i^4 n_{ik} n_{il} / f_k f_l$, yielding

$$\text{Det } m^2 = (\text{Det}\mathbb{N})^2 \prod_i \frac{\Lambda_i^4}{f_i^2}. \quad (48)$$

In the presence of light flat direction, its mass square is given by

$$m_{\text{flat}}^2 \simeq \frac{\Lambda_N^4}{f_{\text{eff}}^2} = \frac{\Lambda_N^4 (\text{Det}\mathbb{N})^2}{\sum_j C_j^2 f_j^2}. \quad (49)$$

Then the mass-square determinant can be written as

$$\begin{aligned} \text{Det } m^2 &= (\text{Det}\mathbb{N})^2 \prod_i \frac{\Lambda_i^4}{f_i^2} = \frac{\Lambda_N^4 f_{\text{eff}}^2}{f_{\text{eff}}^2 f_N^2} (\text{Det}\mathbb{N})^2 \prod_i \frac{\Lambda_i^4}{f_i^2} \\ &= m_{\text{flat}}^2 \frac{\sum_j C_j^2 f_j^2}{f_N^2} \prod_i \frac{\Lambda_i^4}{f_i^2} \sim m_{\text{flat}}^2 \prod_i \frac{N n^2 \Lambda_i^4}{e f_i^2} \\ &\sim m_{\text{flat}}^2 \prod_i m_{\text{heavy}}^2(i), \end{aligned} \quad (50)$$

where $m_{\text{heavy}}(i)$ is the i th heavy axion mass, and we have used $\sum_j C_j^2 f_j^2 / f_N^2 \sim N! n^{2(N-1)} \sim N^N n^{2(N-1)} / e^N$ under the assumption that all f_i are comparable to each other. We then find

$$\frac{m_{\text{heavy}}^2(i)}{m_{\text{flat}}^2} \sim \frac{N n^2 f_{\text{eff}}^2 \Lambda_i^4}{e f_i^2 \Lambda_N^4}, \quad (51)$$

which shows that the heavy axions are heavy enough compared to the flat direction if the anomaly coefficients are aligned to yield $f_{\text{eff}} \gg f_i$, even when Λ_N is comparable to Λ_i ($i = 1, \dots, N-1$).

As in the case with two axions, it is clear that $f_{\text{eff}} \gg f_i$ is not a generic feature, but requires a specific alignment of the anomaly coefficients. Yet, compared to the case of two axions, a notable difference is that the mechanism does not require large anomaly coefficients of $\mathcal{O}(f_{\text{eff}}/f_i)$, but a moderately large number of axions $N \ln N \gtrsim 2 \ln(f_{\text{eff}}/f_i)$ together with the anomaly coefficients $n_{ij} = \mathcal{O}(1)$. To quantify the required degree of alignment, let us introduce an alignment angle as in the case of two axions:

$$\sin \delta\theta \equiv \frac{|\vec{C} \cdot \vec{n}_N|}{|\vec{C}| |\vec{n}_N|} = \frac{|\text{Det}\mathbb{N}|}{|\vec{C}| |\vec{n}_N|}. \quad (52)$$

Then the effective decay constant reads as

$$f_{\text{eff}} = \frac{1}{\sin \delta\theta} \frac{\sqrt{\sum_{i=1}^N C_i^2 f_i^2}}{|\vec{C}| |\vec{n}_N|} \sim \frac{f_i}{|\vec{n}_N| \sin \delta\theta}, \quad (53)$$

implying that we need to align $\delta\theta$ to be small as

$$\delta\theta = \mathcal{O}(f_i/f_{\text{eff}}). \quad (54)$$

This also suggests that the probability for having $f_{\text{eff}} \gg f_i$ under a random choice of the anomaly coefficients in the range $|n_{ij}| \leq n_{\text{max}} = \mathcal{O}(1)$ is given by

$$P(f_{\text{eff}}/f_i) = \mathcal{O}(f_i/f_{\text{eff}}). \quad (55)$$

Note that in the case of two axions we have a similar probability only when the random selection is limited to a specific (economic) region of the anomaly coefficients, for instance the region of (25), while $P(f_{\text{eff}}/f_i) = \mathcal{O}(f_i^2/f_{\text{eff}}^2)$ if one extends the random selection to the general region where all the anomaly coefficients can be comparable to each other.

In view of that the fundamental axion decay constants in string theory are typically in the range $f_i \sim 10^{16}-10^{17}$ GeV [5], while a successful natural inflation compatible with the recent BICEP2 results [1] requires $f_{\text{eff}} \gtrsim 10M_{\text{Pl}}$, we are particularly interested in the *minimal* number of axions which can yield

$$f_{\text{eff}}/f_i = \mathcal{O}(10^2-10^3) \quad \text{for } |n_{ij}| \leq n_{\text{max}} \quad (n_{\text{max}} = 1, 2, 3). \quad (56)$$

According to our discussion above, the corresponding range of N is roughly given by

$$N \ln N \gtrsim 2 \ln(f_{\text{eff}}/f_i). \quad (57)$$

We have performed a numerical analysis to evaluate $P(f_{\text{eff}}/f_i)$ for the three different values of f_{eff}/f_i :

$$f_{\text{eff}}/f_i = 10^2, \quad 5 \times 10^2, \quad 10^3,$$

when

$$\begin{aligned} N &= 8-15 (n_{\text{max}} = 1), & N &= 7-14 (n_{\text{max}} = 2), \\ N &= 5-12 (n_{\text{max}} = 3). \end{aligned}$$

The results are depicted in Table I, which confirms that the probability for the necessary alignment to be achieved under a random choice of the anomaly coefficients is indeed of the order of f_i/f_{eff} .

Before closing this section, let us present a couple of explicit models which achieve an exponentially enhanced effective axion decay constant within the framework discussed above. Our first model is

$$\begin{aligned} V &= \Lambda_1^4 \left[1 - \cos \left(\frac{\phi_1}{f_1} + \frac{n_2 \phi_2}{f_2} \right) \right] \\ &+ \Lambda_2^4 \left[1 - \cos \left(\frac{\phi_2}{f_2} + \frac{n_3 \phi_3}{f_3} \right) \right] + \dots \\ &+ \Lambda_{N-1}^4 \left[1 - \cos \left(\frac{\phi_{N-1}}{f_{N-1}} + \frac{n_N \phi_N}{f_N} \right) \right] \\ &+ \Lambda_N^4 \left[1 - \cos \left(\frac{\phi_N}{f_N} \right) \right], \quad (58) \end{aligned}$$

which is designed to realize a multiple axion monodromy along the inflaton direction. The anomaly coefficient matrix of the model takes the form

TABLE I. The probability to f_i/f_{eff} ratio, $R = P(f_i/f_{\text{eff}})$, for the necessary alignment under 10^5 random choices of the anomaly coefficients. We have considered $N = 8-15$ for $n_{\text{max}} = 1$, $N = 7-14$ for $n_{\text{max}} = 2$, and $N = 5-12$ for $n_{\text{max}} = 3$.

N	f_{eff}/f_i			N	f_{eff}/f_i			N	f_{eff}/f_i		
	100	500	1000		100	500	1000		100	500	1000
8	0.009	0.	0.	7	0.216	0.030	0.	5	0.060	0.	0.
9	0.064	0.	0.	8	0.411	0.210	0.05	6	0.202	0.060	0.
10	0.258	0.020	0.	9	0.466	0.530	0.26	7	0.322	0.230	0.09
11	0.487	0.105	0.01	10	0.542	0.445	0.43	8	0.373	0.225	0.31
12	0.707	0.275	0.12	11	0.512	0.470	0.70	9	0.383	0.370	0.40
13	0.797	0.500	0.41	12	0.519	0.660	0.64	10	0.393	0.390	0.30
14	0.938	0.800	0.56	13	0.585	0.585	0.47	11	0.408	0.370	0.41
15	0.855	0.850	0.77	14	0.530	0.490	0.43	12	0.404	0.355	0.39

$$\mathbb{N} = (n_{ij}) = \begin{pmatrix} 1 & n_2 & & & \\ & 1 & n_3 & & \\ & & 1 & \ddots & \\ & & & \ddots & n_N \\ & & & & 1 \end{pmatrix}, \quad (59)$$

for which

$$\text{Det}\mathbb{N} = 1, \quad |C_i| = \prod_{j=i+1}^N n_j. \quad (60)$$

The resulting effective axion decay constant is given by

$$f_{\text{eff}} = \frac{\sqrt{\sum_{i=1}^N f_i^2 C_i^2}}{\text{Det}\mathbb{N}} = \left(\sum_{i=1}^N \prod_{j=i+1}^N n_j^2 f_i^2 \right)^{1/2} \sim n_2 n_3 \cdots n_N f_1 \quad (61)$$

if $n_i > 1$ and $f_1 \sim f_2 \sim \dots \sim f_N$. In the limit $\Lambda_N \ll \Lambda_i$ ($i = 1, \dots, N-1$), the inflaton direction is determined to be

$$\begin{aligned} \phi_{\text{flat}} \propto \sum_i C_i \phi_i &= \left(\prod_{i=2}^N n_i \right) \phi_1 - \left(\prod_{i=3}^N n_i \right) \phi_2 \\ &+ \cdots - n_N \phi_{N-1} + \phi_N. \end{aligned} \quad (62)$$

This model can be considered as a generalization of the two axion models of [14,15], and realizes a multiple axion monodromy $\prod_{i=2}^N Z_{n_i}$ along the inflaton direction. As a consequence, in order for the N th axion ϕ_N to travel one period along the inflaton direction, i.e. $\Delta\phi_N = 2\pi f_N$, the other axions ϕ_i ($i = 1, 2, \dots, N-1$) should experience a multiple winding as

$$\frac{\Delta\phi_i}{2\pi f_i} = \prod_{j=i+1}^N n_j. \quad (63)$$

In Fig. 2, we depict such multiple monodromy structure for the case of $N = 3$ and $n_2 = n_3 = 2$.

Our second model is

$$\begin{aligned} V &= \Lambda_1^4 \left[1 - \cos \left(\frac{\phi_1}{f_1} + \frac{\phi_2}{f_2} - \frac{\phi_3}{f_3} + \cdots + (-)^N \frac{\phi_N}{f_N} \right) \right] \\ &+ \Lambda_2^4 \left[1 - \cos \left(\frac{\phi_2}{f_2} + \frac{\phi_3}{f_3} - \frac{\phi_4}{f_4} + \cdots + (-)^{N-1} \frac{\phi_N}{f_N} \right) \right] \\ &+ \cdots + \Lambda_{N-2}^4 \left[1 - \cos \left(\frac{\phi_{N-2}}{f_{N-2}} + \frac{\phi_{N-1}}{f_{N-1}} - \frac{\phi_N}{f_N} \right) \right] \\ &+ \Lambda_{N-1}^4 \left[1 - \cos \left(\frac{\phi_{N-1}}{f_{N-1}} + \frac{\phi_N}{f_N} \right) \right] \\ &+ \Lambda_N^4 \left[1 - \cos \left(\frac{\phi_N}{f_N} \right) \right], \end{aligned} \quad (64)$$

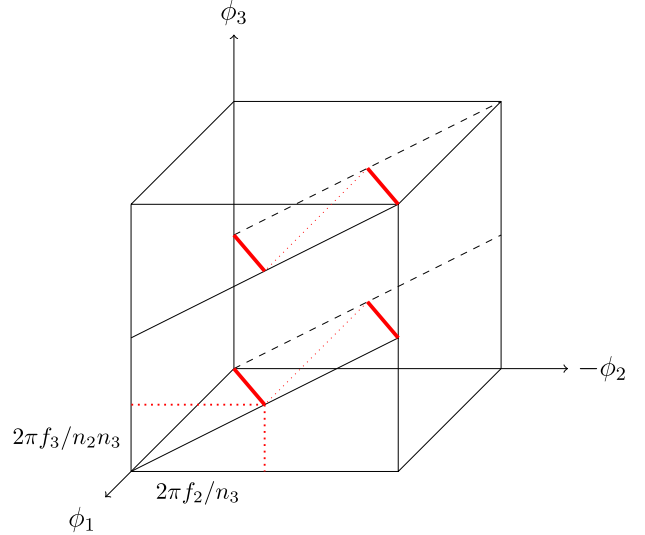


FIG. 2 (color online). Multiple monodromy structure for the three-axion model with $n_2 = n_3 = 2$. The solid red line represents the inflaton direction in the fundamental domain of three axions. Note that $\Delta\phi_2 = 2\pi f_2$ along the inflaton direction requires $\Delta\phi_1 = 2\pi n_2 f_1$, and $\Delta\phi_3 = 2\pi f_3$ requires $\Delta\phi_2 = 2\pi n_3 f_2$. As a result, $\Delta\phi_3 = 2\pi f_3$ along the inflaton direction yields $\Delta\phi_1 = 2\pi n_2 n_3 f_1$.

which is designed to achieve an exponentially enhanced effective decay constant even when all the integer-valued anomaly coefficients are restricted as $|n_{ij}| \leq 1$. In this case, we have

$$\begin{aligned} \text{Det}\mathbb{N} &= 1, \quad C_i^2 = 4^{(N-1-i)} \quad (i = 1, 2, \dots, N-1), \\ C_N^2 &= 1. \end{aligned} \quad (65)$$

Then, assuming that all fundamental decay constants are comparable to each other, the effective decay constant is exponentially enhanced as

$$f_{\text{eff}} \sim \sqrt{\frac{1}{3}(4^{N-1} + 2)} f_i \sim \frac{2^{N-1}}{\sqrt{3}} f_i \quad (66)$$

although we have $|n_{ij}| \leq 1$.

IV. CONCLUSION

Natural inflation provides an attractive framework for large field chaotic inflation which can explain the recent detection of primordial gravitational waves by BICEP2. The KNP alignment mechanism offers an interesting scheme to get a super-Planckian effective axion decay constant $f_{\text{eff}} \gg M_{\text{Pl}}$, which is a necessary component of natural inflation, starting from sub-Planckian fundamental axion decay constants $f_i \ll M_{\text{Pl}}$ of multiple axions. In this paper, we extended the original KNP model with two axions to models with $N > 2$ axions. Compared to the original KNP model, a notable difference is that large

anomaly coefficients of $\mathcal{O}(f_{\text{eff}}/f_i)$ are not required anymore if the number of axions is moderately large as $N \ln N \gtrsim 2 \ln(f_{\text{eff}}/f_i)$. With such N , the effective decay constant can be exponentially enhanced as $f_{\text{eff}}/f_i \sim \sqrt{N!} n^{N-1}$ for n denoting the typical size of the integer-valued anomaly coefficients, and the probability for the necessary alignment to be achieved under a random choice of the anomaly coefficients is of the order of f_i/f_{eff} .

The structure of our setup is rich enough to realize a variety of different possibilities. For instance, it can realize

a multiple axion monodromy $\prod_{i=2}^N Z_{n_i}$ yielding $f_{\text{eff}}/f_i \sim \prod_{i=2}^N n_i$. The setup can also give rise to an exponentially enhanced effective axion decay constant as $f_{\text{eff}}/f_i \sim 2^{N-1}$, even when all the integer-valued anomaly coefficients are restricted as $|n_{ij}| \leq 1$.

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- [1] P. A. R. Ade *et al.* (BICEP2 Collaboration), *Phys. Rev. Lett.* **112**, 241101 (2014).
- [2] D. H. Lyth, *Phys. Rev. Lett.* **78**, 1861 (1997).
- [3] K. Freese, J. A. Frieman, and A. V. Olinto, *Phys. Rev. Lett.* **65**, 3233 (1990); F. C. Adams, J. R. Bond, K. Freese, J. A. Frieman, and A. V. Olinto, *Phys. Rev. D* **47**, 426 (1993).
- [4] K. Freese and W. H. Kinney, [arXiv:1403.5277](https://arxiv.org/abs/1403.5277).
- [5] K. Choi and J. E. Kim, *Phys. Lett.* **154B**, 393 (1985); **156B**, 452(E) (1985); P. Svrcek and E. Witten, *J. High Energy Phys.* **06** (2006) 051; T. Banks, M. Dine, P. J. Fox, and E. Gorbatov, *J. Cosmol. Astropart. Phys.* **06** (2003) 001.
- [6] J. E. Kim, H. P. Nilles, and M. Peloso, *J. Cosmol. Astropart. Phys.* **01** (2005) 005.
- [7] N. Arkani-Hamed, H.-C. Cheng, P. Creminelli, and L. Randall, *Phys. Rev. Lett.* **90**, 221302 (2003).
- [8] S. Dimopoulos, S. Kachru, J. McGreevy, and J. G. Wacker, *J. Cosmol. Astropart. Phys.* **08** (2008) 003.
- [9] A. R. Liddle, A. Mazumdar, and F. E. Schunck, *Phys. Rev. D* **58**, 061301 (1998); E. J. Copeland, A. Mazumdar, and N. J. Nunes, *Phys. Rev. D* **60**, 083506 (1999); P. Kanti and K. A. Olive, *Phys. Lett. B* **464**, 192 (1999).
- [10] E. Silverstein and A. Westphal, *Phys. Rev. D* **78**, 106003 (2008); L. McAllister, E. Silverstein, and A. Westphal, *Phys. Rev. D* **82**, 046003 (2010).
- [11] F. Marchesano, G. Shiu, and A. M. Uranga, [arXiv:1404.3040](https://arxiv.org/abs/1404.3040); A. Hebecker, S. C. Kraus, and L. T. Witkowski, [arXiv:1404.3711](https://arxiv.org/abs/1404.3711).
- [12] N. Kaloper and L. Sorbo, *Phys. Rev. Lett.* **102**, 121301 (2009); N. Kaloper, A. Lawrence, and L. Sorbo, *J. Cosmol. Astropart. Phys.* **03** (2011) 023.
- [13] K. Harigaya and M. Ibe, [arXiv:1404.3511](https://arxiv.org/abs/1404.3511) [*Phys. Rev. Lett.* (to be published)].
- [14] S.-H. H. Tye and S. S. C. Wong, [arXiv:1404.6988](https://arxiv.org/abs/1404.6988).
- [15] I. Ben-Dayan, F. G. Pedro, and A. Westphal, [arXiv:1404.7773](https://arxiv.org/abs/1404.7773).
- [16] A. Chatzistavrakidis, E. Erfani, H. P. Nilles, and I. Zavala, *J. Cosmol. Astropart. Phys.* **09** (2012) 006.