QCD axion as a bridge between string theory and flavor physics

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We construct a string-inspired model, motivated by the flavored Peccei-Quinn (PQ) axions, as a useful bridge between flavor physics and string theory. The key feature is two anomalous gauged $U(1)$ symmetries, responsible for both the fermion mass hierarchy problem of the standard model and the strong CP problem, that combine string theory with flavor physics and severely constrain the form of the F- and D-term contributions to the potential. In the context of supersymmetric moduli stabilization we stabilize the size moduli with positive masses while leaving two axions massless and one axion massive. We demonstrate that, while the massive gauge bosons eat the two axionic degrees of freedom, two axionic directions survive to low energies as the flavored PQ axions.

I. INTRODUCTION

The standard model (SM) as an effective theory has been successful in describing phenomena until now, but it suffers from theoretical problems (inclusion of gravity in gauge theory, instability of the Higgs potential, the SM fermion mass hierarchies and their mixing patterns with the CP violating phases, the strong CP problem [1], etc.) and cosmological issues (dark matter, inflation, cosmological constant, etc.). It is widely believed that the SM should be extended to a more fundamental underlying theory. If nature is stringy, string theory should give insight into all such fundamental problems. Therefore, we can anticipate that there may exist some correlation between string theory as a fundamental theory and low energy flavor physics.

Reference [2] used a superpotential for unifying flavor and strong CP problems, the so-called flavored Peccei-Quinn (PQ) symmetry model, in a way that no axionic domain wall problem occurs. In this paper we construct an explicit string-inspired model, motivated by the flavored PQ axions, as a useful bridge between flavor physics and string theory. The key features of the model can be present in type IIB compactification. The crucial one is two anomalous gauged $U(1)$ symmetries that combine string theory with flavor physics, and severely constrain the form of the F- and D-term contributions to the potential. We show how supersymmetric moduli stabilization with three fixed size moduli, one fixed axionic partner and two unfixed axions can be realized. We illustrate that the model admits metastable vacuum with spontaneously broken supersymmetry (SUSY) and a nearly vanishing positive vacuum energy, resulting from the positive contributions to the potential associated with the gauge symmetry of the theory, the so-called D-terms. In addition, we illustrate how to achieve phenomenologically nontrivial vacuum

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expectation value (VEV) directions of flavon fields. Finally, we demonstrate that, while the massive gauge bosons eat the axionic degree of freedoms, two axionic directions survive to low energies as the flavored PQ axions [2].

II. THE MODEL

Below the scale where the dilaton and complex structure moduli are stabilized through fluxes [3], we consider the low-energy Kahler potential K and superpotential W for the Kahler moduli and matter superfields invariant under gauged $U(1)_X$ symmetry

$$K = -M_P^2 \ln \left\{ (T + \bar{T}) \prod_{i=1}^{2} \left(T_i + \bar{T}_i - \frac{\delta_i^{\text{GS}}}{16\pi^2} V_{X_i} \right) \right\}$$
$$+ \sum_{i=1}^{2} Z_i \Phi_i^\dagger e^{-X_i V_{X_i}} \Phi_i + \sum_k Z_k |\varphi_k|^2 + \cdots \qquad (1)$$

$$W = W_Y + W_v + W_0 + W(T) \qquad (2)$$

which is appropriate for toroidal orientifold, where $M_P = m_{\text{P}}/\sqrt{8\pi} = 2.4 \times 10^{18}$ GeV is the reduced Planck mass, and dots stand for higher order terms. The first term in Eq. (1) has a no-scale symmetry up to perturbative corrections from string theory. Note that the Kahler moduli do not appear in the superpotential at tree level, therefore they are not fixed by the fluxes. From the Kahler potential and superpotential, we schematically obtain the low-energy effective Lagrangian

$$\mathcal{L} \supset \frac{1}{2} K_{T\bar{T}} \partial_\mu T \partial^\mu \bar{T} + \frac{1}{2} K_{T_i \bar{T}_i} \partial_\mu T_i \partial^\mu \bar{T}_i - V$$
$$+ \mathcal{L}(\Phi_i, \varphi_i, \ldots). \qquad (3)$$

Here the kinetic terms for the axionic and size moduli do not mix in perturbation theory, due to the axionic shift

symmetry, where any nonperturbative violations are small enough to be irrelevant. The Kahler metric in Eq. (3) is given by

$$K_{IJ} = M_P^2 \begin{pmatrix} (T + \bar{T})^{-2} & 0 & 0 \\ 0 & (T_1 + \bar{T}_1)^{-2} & 0 \\ 0 & 0 & (T_2 + \bar{T}_2)^{-2} \end{pmatrix} \tag{4}$$

for $V_{X_i} = 0$, in which I, J stand for T, T_1, and T_2. The Kahler moduli in K of Eq. (1) control the overall size of the compact space,

$$T = \frac{\tau}{2} + i\theta, \quad T_i = \frac{\tau_i}{2} + i\theta_i \quad \text{with} \quad i = 1, 2. \tag{5}$$

As can be seen from the Kahler potential above, the relevant fields participating in the four-dimensional Green-Schwarz (GS) mechanism are the $U(1)_{X_i}$ charged chiral matter superfields Φ_i, the vector fields V_{X_i} of the anomalous $U(1)_{X_i}$, and the Kahler moduli T_i. The matter superfields in K consist of all the scalar fields Φ_i that are not moduli and do not have Planck sized VEVs, and the chiral matter fields φ_k are neutral under the $U(1)_X$ symmetry. δ_i^{GS} stand for the coefficients of the mixed $U(1)_{X_i}$-$SU(3)_c$-$SU(3)_c$ color anomalies which are canceled by the GS mechanism, $\delta_i^{\text{GS}} \delta^{ab} = 2\sum_{\psi_i} X_i \text{Tr}[t^a t^b]$, where t^a are the generators of the representation of $SU(3)$ to which ψ belongs and the sum runs over all Dirac fermions ψ with X-charge. We take, for simplicity, the normalization factors $Z_i = Z_k = 1$, and the holomorphic gauge kinetic function on the Kahler moduli

$$T_i = \frac{1}{g_{X_i}^2} + i\frac{a_{T_i}}{8\pi^2}, \tag{6}$$

where g_{X_i} are the four-dimensional gauge couplings of $U(1)_{X_i}$. Actually, gaugino masses require a nontrivial dependence of the holomorphic gauge kinetic function on the Kahler moduli. This dependence is generic in most of the models of $\mathcal{N} = 1$ SUGRA derived from extended supergravity and string theory [4]. Vector multiplets V_{X_i} in Eq. (1) are the $U(1)_{X_i}$ gauge superfields including gauge bosons A_i^μ.

In the Kahler potential and superpotential in Eqs. (1) and (2) we have introduced two anomalous gauged $U(1)_X \equiv U(1)_{X_1} \times U(1)_{X_2}$ with anomalies canceled via exchange of two Kahler-axion fields θ_i and two kinds of scalar fields Φ_i with charges X_i, in order to explain both the fermion mass hierarchy problem of the SM and the strong CP problem [5]. The model group $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_X \times U(1)_R$ we are interested in may be realized in a four-stack model $U(3) \times U(2) \times U(1) \times U(1)$ on D-branes where the gauged $U(1)$s are genericaly anomalous [6].

Hypercharge $U(1)_Y$ is the unique anomaly-free linear combination of the four $U(1)$s. The other combinations contribute to $U(1)_X$ and a gauged $U(1)_R$ [7] which contains an R-symmetry as a subgroup: {flavor matter fields $\rightarrow e^{i\xi/2}$ flavor matter fields} and {driving fields $\rightarrow e^{i\xi}$ driving fields}, with $W \rightarrow e^{i\xi} W$, whereas flavon and Higgs fields remain invariant, and an axionic shift. In addition, one can introduce a non-Abelian discrete flavor symmetry, such as [8], to describe flavor mixing pattern, which can be realized in field theories on orbifolds [9]. (We will not discuss them here.) W_0 is the constant value of the flux superpotential at its minimum. $W(T)$ is a certain nonperturbative term, which is introduced to stabilize the Kahler moduli. Although $W(T)$ in Eq. (2) is absent at tree level, the Kahler moduli appear nonperturbatively in the superpotential through brane instantons or gaugino condensation [10]. The superpotential W_v dependent on the driving fields, invariant under $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_X \times A_4$, is given at leading order by [2]

$$\begin{aligned} W_v &= \Phi_0^T (\tilde{\mu}\Phi_T + \tilde{g}\Phi_T\Phi_T) + \Phi_0^S (g_1\Phi_S\Phi_S + g_2\tilde{\Theta}\Phi_S) \\ &+ \Theta_0 (g_3\Phi_S\Phi_S + g_4\Theta\Theta + g_5\Theta\tilde{\Theta} + g_6\tilde{\Theta}\tilde{\Theta}) \\ &+ g_7\Psi_0(\Psi\tilde{\Psi} - \mu_\Psi^2), \end{aligned} \tag{7}$$

where $\tilde{\mu}$ is a dimensionful parameter and $\tilde{g}$, $g_{1,...,7}$ are dimensionless coupling constants. The details of the A_4 group are shown in the Appendix. The non-Abelian discrete flavor symmetry A_4 on W_v is properly imposed, apart from the usual two Higgs doublets $H_{u,d}$ responsible for electroweak symmetry breaking, which are invariant under A_4 (i.e. flavor singlets), on two new types of scalar multiplets: flavon fields, responsible for the spontaneous breaking of the flavor symmetry, Φ_T, Φ_S, Θ, $\tilde{\Theta}$, Ψ, $\tilde{\Psi}$ that are $SU(2)$-singlets; and driving fields Φ_0^T, Φ_0^S, Θ_0, Ψ_0 that are associated to a nontrivial scalar potential in the symmetry breaking sector. We take the flavon fields Φ_T, Φ_S to be A_4 triplets, and Θ, $\tilde{\Theta}$, Ψ, $\tilde{\Psi}$ to be A_4 flavor singlets, respectively, that are $SU(2)$-singlets, and driving fields Φ_0^T, Φ_0^S to be A_4 triplets and Θ_0, Ψ_0 to be an A_4 singlet. In addition, there is flavored PQ symmetry $U(1)_X$ which is mainly responsible for the fermion mass hierarchy of the SM, which is composed of two anomalous *gauged* symmetries $U(1)_{X_1} \times U(1)_{X_2}$ generated by the charges X_1 and X_2: $\Phi_1 = \{\Phi_S, \Theta, \tilde{\Theta}\}$, $\Phi_2 = \{\Psi, \tilde{\Psi}\}$ are $U(1)_{X_1}$ and $U(1)_{X_2}$-charged chiral superfields, respectively. The Yukawa superpotential W_Y could be appropriately arranged under the $A_4 \times U(1)_X$ as in Ref. [2], where the seesaw mechanism [11] is embedded, and the fermion Yukawa couplings are visualized as functions of the gauge singlet flavon fields scaled by a cutoff proportional to string scale.

Under the $U(1)_X$ gauge transformation $V_{X_i} \rightarrow V_{X_i} + i(\Lambda_i - \bar{\Lambda}_i)$, the matter and Kahler moduli superfields transform as

$$\Phi_i \to e^{iX_i\Lambda_i}\Phi_i, \qquad T_i \to T_i + i\frac{\delta_i^{GS}}{16\pi^2}\Lambda_i, \qquad (8)$$

where $\Lambda(\bar{\Lambda}_i)$ are (anti)chiral superfields parametrizing $U(1)_{X_i}$ transformations on the superspace. So the axionic moduli θ_i and matter axions A_i have shift symmetries

$$\theta_i \to \theta_i - \frac{\delta_i^{GS}}{16\pi^2}\xi_i, \qquad A_i \to A_i + X_i v_i \xi_i, \qquad (9)$$

where $\xi_i = \mathrm{Re}\Lambda_i|_{\theta=\bar{\theta}=0}$ and $\Phi_i|_{\theta=\bar{\theta}=0} = \frac{1}{\sqrt{2}}e^{i\frac{A_i}{v_i}}(v_i + h_i)$ (here v_i and h_i being the VEVs and Higgs bosons of scalar components, respectively), with the gauge transformation

$$A_i^\mu \to A_i^\mu - \partial^\mu \xi_i. \qquad (10)$$

Then the anomaly generated by the triangle graph is canceled by the diagram in which the anomalous $U(1)_X$ mixes with the axionic moduli, which in turn couples to a multiple of the QCD instanton density $\mathrm{Tr}(G^{\mu\nu}\tilde{G}_{\mu\nu})$ for the gauge group in the compactification. So the axion decay constant depends on the Kahler metric, and in particular on where the moduli are stabilized, which will be shown in Eq. (40).

III. A REALISTIC MODULI STABILIZATION

Since the three moduli appear in the Kahler potential Eq. (2), by solving the F-term equations we are going to stabilize three size moduli with positive masses while leaving two axions massless and one axionic-partner massive through an effective superpotential $W(T)$. The two massless axion directions will be gauged by the $U(1)$ gauge interactions associated with D-branes, and the gauged flat directions of the F-term potential are removed through the Stuckelberg mechanism. The F-term scalar potential has the form

$$V_F = e^{K/M_P^2}\left\{ K^{I\bar{J}}D_I W \bar{D}_{\bar{J}}\bar{W} - \frac{3}{M_P^2}|W|^2 + K^{i\bar{j}}D_i W \bar{D}_{\bar{j}}\bar{W} \right\}, \qquad (11)$$

where $K^{I\bar{J}}$ ($K^{i\bar{j}}$) is the inverse Kahler metric, and I, J stand for T, T_i, and i, j for the bosonic components of the superfields Φ_i, φ_i, and the Kahler covariant derivative and Kahler metric are defined as

$$D_I W \equiv \partial_I W + \frac{W}{M_P^2}\partial_I K, \qquad K_{I\bar{J}} \equiv \partial_I \partial_{\bar{J}} K. \qquad (12)$$

To accomplish our purpose, we take a racetrack-type [12] superpotential as an effective superpotential

$$W(T) = A(\Phi_i)e^{-a(T+T_1+T_2)} + B(\Phi_i)e^{-b(T+T_1+T_2)}, \qquad (13)$$

where $A(\Phi_i)$ and $B(\Phi_i)$ are analytic functions of Φ_i transforming under $U(1)_{X_i}$ as

$$A(\Phi_i) \to A(\Phi_i)e^{\frac{i}{16\pi^2}\frac{a}{16\pi^2}(\delta_1^{GS}\Lambda_1 + \delta_2^{GS}\Lambda_2)},$$

$$B(\Phi_i) \to B(\Phi_i)e^{\frac{i}{16\pi^2}\frac{b}{16\pi^2}(\delta_1^{GS}\Lambda_1 + \delta_2^{GS}\Lambda_2)}, \qquad (14)$$

and invariant under the other gauge group. Then the scalar potential of the fields $\rho_{(i)}(= \tau_{(i)}/2)$ has local minimum at σ_0, σ_i which is supersymmetric, i.e.,

$$W(\sigma_0, \sigma_i) = 0, \qquad D_T W(\sigma_0, \sigma_i) = D_{T_i}W(\sigma_0, \sigma_i) = 0, \qquad (15)$$

and Minkowski, i.e., $V_F(\sigma_0, \sigma_i) = 0$, where $\sigma_0 = \sigma_i = \frac{1}{a-b}\ln(\frac{aA_0}{bB_0})$. W_0 is fine-tuned as

$$W_0 = -A_0\left(\frac{aA_0}{bB_0}\right)^{-3\frac{a}{a-b}} - B_0\left(\frac{aA_0}{bB_0}\right)^{-3\frac{b}{a-b}}, \qquad (16)$$

where $A_0(B_0)$ are constant values of $A(\Phi_i)(B(\Phi_i))$ at a set of VEVs $\langle\Phi_i\rangle$ that cancel all the D-terms, including the anomalous $U(1)_{X_i}$. Here the constants $W(T)$ is not analytic at the VEVs $\langle\Phi_i\rangle$ where the moduli are stabilized.

The F-term equations $D_T W = D_{T_i}W = 0$ provide $\tau = \tau_i$, and lead to

$$aAe^{-3\frac{a\tau}{2}}e^{-ia\theta^{st}} + bBe^{-3\frac{b\tau}{2}}e^{-ib\theta^{st}}$$
$$+ \frac{W_0 + Ae^{-3\frac{a\tau}{2}}e^{-ia\theta^{st}} + Be^{-3\frac{b\tau}{2}}e^{-ib\theta^{st}}}{\tau} = 0 \qquad (17)$$

for $V_{X_i} = 0$, where $\theta^{st} \equiv \theta + \theta_1 + \theta_2$. This shows that the three size moduli (τ, τ_i) and one axionic direction θ^{st} are fixed, while the two axionic directions ($\theta_1^{st} \equiv \theta - \theta_1$ and $\theta_2^{st} \equiv \theta - \theta_2$) are independent of the above equation. So, without loss of generality, we rebase the superfields T with $\theta^{st} = \mathrm{Im}[T]$ and T_i with $\theta_i^{st} = \mathrm{Im}[T_i]$ as

$$T_{(i)} = \tau_{(i)}/2 + i\theta_{(i)} \to T_{(i)} = \tau_{(i)}/2 + i\theta_{(i)}^{st}. \qquad (18)$$

Then from the F-term scalar potential the masses of the fields $\rho_{(i)}$ and θ^{st}, $m_{\tau_{(i)}}^2 = \frac{1}{2}K^{T\bar{T}}\partial_T\partial_{\bar{T}}V_F|_{T=\bar{T}=\sigma_0}$ and $m_{\theta^{st}}^2 = \frac{1}{2}K^{T\bar{T}}\partial_{\theta^{st}}\partial_{\theta^{st}}V_F|_{T=\bar{T}=\sigma_0}$, respectively, are obtained as follows:

$$m_{\tau_{(i)}}^2 = \frac{3\ln(\frac{aA_0}{bB_0})}{M_P^4(a-b)}$$
$$\times\left\{ A_0 a^2\left(\frac{aA_0}{bB_0}\right)^{-3\frac{a}{a-b}} + B_0 b^2\left(\frac{aA_0}{bB_0}\right)^{-3\frac{b}{a-b}} \right\}^2, \qquad (19)$$

$$m_{\theta^{si}}^2 = \frac{3W_0}{M_P^4}\left\{-A_0 a^3\left(\frac{aA_0}{bB_0}\right)^{-3\frac{a}{a-b}} - B_0 b^3\left(\frac{aA_0}{bB_0}\right)^{-3\frac{b}{a-b}}\right\}$$
$$+\frac{6\ln\left(\frac{aA_0}{bB_0}\right)}{M_P^4(a-b)}$$
$$\times\left\{-A_0 B_0(a-b)^2\left(\frac{aA_0}{bB_0}\right)^{-3\frac{a-b}{a-b}}\left(\frac{a^2-b^2}{2\ln\left(\frac{aA_0}{bB_0}\right)}+ab\right)\right\}. \tag{20}$$

Here the mass squared of the size moduli fields $\rho_{(i)}$ at the minimum is simply given by $m_{\tau_{(i)}}^2 = 3\sigma_0|W_{TT}(\sigma_0)|^2/M_P^4$ where $W_{TT} = \partial^2 W/(\partial T)^2$. Note that the gravitino mass in this supersymmetric Minkowski minimum vanishes. With the conditions $a < 0$, $b > 0$ ($|a| < |b|$) and $A_0 > 0$, $B_0 < 0$ we obtain positive values of masses. Here a, b are constants, while A_0, B_0 are constants in M_P^3 units. For a simple choice of parameters, $A_0 = -B_0 = 0.01$, $a = -2\pi/100$ and $b = 2\pi/90$, one has $m_\tau \simeq 1.7 \times 10^{14}$ GeV and $m_{\theta_{si}} \simeq 2.0 \times 10^{14}$ GeV.

IV. SUPERSYMMETRY BREAKING

As discussed in the Kallosh-Linde model [12], supersymmetry is unbroken so far in the vacuum states corresponding to the minimum of the potential with $V = 0$. As will be shown later, the existence of Fayet-Iliopoulos (FI) terms ξ_i^{FI} for the corresponding $U(1)_{X_i}$ implies the existence of uplifting potential which makes a nearly vanishing cosmological constant and induces SUSY breaking. A small perturbation ΔW to the superpotential [12,13] is introduced in order to determine SUSY breaking scale. Then the minimum of the potential is shifted from zero to a slightly negative value at $\sigma_0 + \delta\rho$, $\sigma_i + \delta\rho_i$ by the small constant ΔW. The resulting F-term potential has a supersymmetric anti–de Sitter (AdS) minimum and consequently the depth of this minimum is given in terms of $W(\sigma_0 + \delta\rho, \sigma_i + \delta\rho_i) \simeq \Delta W + \mathcal{O}(\Delta W)^2$ by

$$V_{\mathrm{AdS}} \simeq -\frac{3}{M_P^2}\frac{(\Delta W)^2}{8\sigma_0\sigma_1\sigma_2} = -\frac{3}{8M_P^2}\left(\frac{a-b}{\ln\frac{aA_0}{bB_0}}\right)^2(\Delta W)^2, \tag{21}$$

where $\Delta W = \langle W\rangle_{\mathrm{AdS}}$ is the value of the superpotential at the AdS minimum. At the shifted minimum SUSY is preserved, i.e. $D_T W(\sigma_0 + \delta\rho) = 0$ and $D_{T_i}W(\sigma_i + \delta\rho_i) = 0$, leading to $W_T(\sigma_0 + \delta\rho) = W_{T_i}(\sigma_i + \delta\rho_i) \simeq 3\Delta W/2\sigma_0$. At this new minimum the displacements $\delta\rho = \delta\rho_i$ are obtained as

$$\delta\rho_{(i)} \simeq \frac{3\Delta W}{2\sigma_0 W_{TT}(\sigma_0)}$$
$$= \frac{3(a-b)\Delta W}{2\ln\left(\frac{aA_0}{bB_0}\right)\left\{A_0 a^2\left(\frac{aA_0}{bB_0}\right)^{\frac{-3a}{a-b}} + B_0 b^2\left(\frac{aA_0}{bB_0}\right)^{\frac{-3b}{a-b}}\right\}}. \tag{22}$$

After adding the uplifting potentials SUSY is broken and then the gravitino in the uplifted minimum acquires a mass

$$m_{3/2} \simeq \frac{|\Delta W|}{M_P^2}\left(\frac{a-b}{2\ln\frac{aA_0}{bB_0}}\right)^{\frac{1}{2}}. \tag{23}$$

The uplifting of the AdS minimum to the dS minimum can be achieved by considering nontrivial fluxes for the gauge fields living on the D7 branes [14] which can be identified as field-dependent FI D-terms in the $\mathcal{N} = 1$, 4D effective action [15]. The uplifting terms can be parametrized as $\Delta V_i = \frac{1}{2}(\xi_i^{\mathrm{FI}})^2 g_{X_i}^2 \simeq |V_{\mathrm{AdS}}|(\sigma_0/\rho_i)^3$ [14] such that the value of the potential at the new minimum becomes equal to the observed value of the cosmological constant. So, as will be shown later, the anomalous FI terms cannot be canceled, and act as uplifting potential. Expanding the Kahler potential K in components, the term linear in V_{X_i} produces the FI factors $\xi_i^{\mathrm{FI}} = \frac{\partial K}{\partial V_{X_i}}|_{V_{X_i}=0}\Delta\rho_i$ as

$$\xi_i^{\mathrm{FI}} = M_P^2\frac{\delta_i^{\mathrm{GS}}}{16\pi^2\tau}\Delta\rho_i. \tag{24}$$

Here the displacements $\Delta\rho_i \equiv \rho_i - \sigma_0$ in the moduli fields are induced by the uplifting terms,

$$\Delta\rho_i \simeq \frac{6M_P^2|V_{\mathrm{AdS}}|}{W_{TT}^2(\sigma_0)}, \tag{25}$$

which are achieved by $\partial_{\rho_i}(V_F + \Delta V_i)|_{\sigma_i+\delta\rho_i} = 0$. Since the uplifting terms by $\Delta\rho_i$ making the dS minimum induce SUSY breaking, all particles whose mass is protected from supersymmetry become massive. With our choice of parameters, the gravitino mass being of order 10 TeV implies $|\Delta W| \simeq 10^{-14}M_P^3$, and which in turn means that the FI terms proportional to $|V_{\mathrm{AdS}}|/m_\tau^2$ are expected to be strongly suppressed.

Setting to zero from the beginning the SM matter fields $\{q^c, \ell, H_u, \ldots\}$, with the almost vanishing cosmological constant for the remaining fields the gravitino mass $m_{3/2}$ is directly related to the scale of supersymmetry breaking, $|F|^2 - 3m_{3/2}^2 M_P^2 + D_{X_i}^2/2 \approx 0$, implying that the F- and D-term potentials should vanish in the limit $m_{3/2}$ going to zero and some of them should scale as $m_{3/2}$ at the minimum. In the global SUSY limit, i.e. $M_P \to \infty$, the relevant F-term potential is written as

$$V_F^{\text{global}} = \left| \frac{2g_1}{\sqrt{3}} (\Phi_{S1}\Phi_{S1} - \Phi_{S2}\Phi_{S3}) + g_2\Phi_{S1}\tilde{\Theta} \right|^2$$

$$+ \left| \frac{2g_1}{\sqrt{3}} (\Phi_{S2}\Phi_{S2} - \Phi_{S1}\Phi_{S3}) + g_2\Phi_{S3}\tilde{\Theta} \right|^2$$

$$+ \left| \frac{2g_1}{\sqrt{3}} (\Phi_{S3}\Phi_{S3} - \Phi_{S1}\Phi_{S2}) + g_2\Phi_{S2}\tilde{\Theta} \right|^2$$

$$+ |g_3(\Phi_{S1}\Phi_{S1} + 2\Phi_{S2}\Phi_{S3}) + g_4\Theta^2 + g_5\Theta\tilde{\Theta} + g_6\tilde{\Theta}^2|^2$$

$$+ |g_7(\Psi\tilde{\Psi} - \mu_{\tilde{\Psi}}^2)|^2 + |g_7|^2|\Psi_0|^2(|\Psi|^2 + |\tilde{\Psi}|^2) + \sum_{i=\text{the others}} \left| \frac{\partial W_\nu}{\partial z_i} \right|^2, \tag{26}$$

and the D-term potential, obtained by the introduction of two FI D-terms $\mathcal{L}_i^{\text{FI}} = -g_{X_i}\xi_i^{\text{FI}}D_{X_i}$, is given by

$$V_D^{\text{global}} = \frac{|X_1|^2 g_{X_1}^2}{2} \left(\frac{\xi_1^{\text{FI}}}{|X_1|} - |\Phi_S|^2 - |\Theta|^2 - |\tilde{\Theta}|^2 \right)^2$$

$$+ \frac{|X_2|^2 g_{X_2}^2}{2} \left(\frac{\xi_2^{\text{FI}}}{|X_2|} - |\Psi|^2 + |\tilde{\Psi}|^2 \right)^2 \tag{27}$$

with $D_{X_i} = g_{X_i}(\xi_i^{\text{FI}} - \sum_i X_i |\Phi_i|^2)$, where $\xi_i^{\text{FI}} = 2E_i/\tau_i$ are constant parameters with dimensions of mass squared and here E_i are measure of the strength of the fluxes for the gauge fields living on the D7 branes [14]. Since SUSY is preserved after the spontaneous symmetry breaking of $U(1)_X \times A_4$, the scalar potential in the limit $M_P \to \infty$ vanishes at its ground states, i.e., vanishing F-terms must have also vanishing D-terms. Consequently, the VEVs of the flavon fields are from the minimization conditions of the F-term scalar potential: the phenomenologically nontrivial solutions [2]

$$\langle\Phi_S\rangle = \frac{1}{\sqrt{2}}(v_S, v_S, v_S), \qquad \langle\Theta\rangle = \frac{v_\Theta}{\sqrt{2}},$$

$$\langle\Psi\rangle = \langle\tilde{\Psi}\rangle = \frac{v_\Psi}{\sqrt{2}}, \tag{28}$$

as well as a set of trivial solutions

$$\langle\Phi_S\rangle = (0,0,0), \qquad \langle\Theta\rangle = 0, \qquad \langle\Psi\rangle = \langle\tilde{\Psi}\rangle = \frac{v_\Psi}{\sqrt{2}}. \tag{29}$$

Then the two supersymmetric solutions are taken by the D-flatness conditions, respectively, for (i) the phenomenologically viable case

$$\xi_1^{\text{FI}} = |X_1|(\langle|\Phi_S|^2\rangle + \langle|\Theta|^2\rangle), \qquad \xi_2^{\text{FI}} = 0, \qquad \langle\Psi\rangle = \langle\tilde{\Psi}\rangle, \tag{30}$$

and (ii) the phenomenologically trivial case

$$\xi_1^{\text{FI}} = \langle\Phi_S\rangle = \langle\Theta\rangle = 0, \qquad \xi_2^{\text{FI}} = 0, \qquad \langle\Psi\rangle = \langle\tilde{\Psi}\rangle, \tag{31}$$

both of which indicate that the VEVs of the flavon fields strictly depend on the moduli stabilization, particularly on the VEVs of the fluxes E_i in the FI terms [14]. So it seems hard for the first case (i) to stabilize $|\Phi_i|$ at large VEVs $\sim \mathcal{O}(10^{12})$ GeV. There is a tension between $\langle\Phi_i\rangle = 0$ and $\langle\xi_i^{\text{FI}}\rangle \neq 0$ which is possible as long as E_i are below the string scale. Therefore it is imperative that, in order for the D-terms to act as uplifting potential, the F-terms have to necessarily break SUSY. In order for the solution in Eq. (29) to be phenomenologically nontrivial, we destabilize $\Phi_1 = \{\Phi_S, \Theta\}$ and $\Phi_2 = \{\Psi, \tilde{\Psi}\}$ by their tachyonic SUSY masses to develop v_S, v_Θ, $v_\Psi(v_{\tilde{\Psi}})$ comparable with seesaw and QCD axion window scales [2], while keeping $\langle\tilde{\Theta}\rangle = 0$ for the scalar field $\tilde{\Theta}$ with $m_{\tilde{\Theta}}^2 > 0$. The phenomenologically viable VEVs of the flavon fields can be determined by considering both the SUSY breaking effect which lift up the flat directions and supersymmetric next-to-leading order terms (see the origin of this argument [16]) invariant under $A_4 \times U(1)_X$. The supersymmetric next-to-leading order terms are given by

$$\Delta W_\nu \simeq \frac{\alpha}{m_P} \Psi\tilde{\Psi}(\Phi_T\Phi_0^T)_1 + \frac{\beta}{m_P}(\Phi_0^S\Phi_T)_1\Theta\Theta$$

$$+ \frac{1}{m_P}\{\gamma_1(\Phi_S\Phi_S)_1(\Phi_T\Phi_0^S)_1 + \gamma_2(\Phi_S\Phi_S)_{1'}(\Phi_T\Phi_0^S)_{1''}$$

$$+ \gamma_3(\Phi_S\Phi_S)_{1''}(\Phi_T\Phi_0^S)_{1'}\}, \tag{32}$$

where α, β, and $\gamma_{1,2,3}$ are real-valued constants being of order unities. Note that here, since we are considering the phenomenologically nontrivial solutions as in Eq. (28), operators including $\tilde{\Theta}$, $(\Phi_S\Phi_S)_{3s}$, $(\Phi_S\Phi_T)_{3s}$, and $(\Phi_S\Phi_T)_{3a}$ are neglected in ΔW_ν. Since soft SUSY-breaking terms are already present at the scale relevant to flavor dynamics, the scalar potentials for $\Psi(\tilde{\Psi})$ and $\Phi_S(\Theta)$ at leading order read

$$V(\Phi_S, \Theta) \simeq \beta_1 m_{3/2}^2 |\Phi_S|^2 + \beta_2 m_{3/2}^2 |\Theta|^2 + \frac{v_T^2 |\beta \Theta^2 + \gamma \Phi_S^2|^2}{2m_P^2},$$

$$V(\Psi, \tilde{\Psi}) \simeq \alpha_1 m_{3/2}^2 |\Psi|^2 + \alpha_2 m_{3/2}^2 |\tilde{\Psi}|^2 + |\alpha|^2 \frac{v_T^2 |\Psi|^2 |\tilde{\Psi}|^2}{2m_P^2},$$

$$(33)$$

leading to the PQ breaking scales

$$\mu_\Psi^2 = \frac{v_\Psi v_{\tilde{\Psi}}}{2} = \frac{2\sqrt{\alpha_1 \alpha_2}}{|\alpha|^2} \left(\frac{m_{3/2}}{v_T} m_P \right)^2, \qquad (34)$$

$$v_S^2 = \frac{2\beta_1 \kappa^2}{\gamma(\beta + \gamma)} \left(\frac{m_{3/2}}{v_T} m_P \right)^2 = \kappa^2 v_\Theta^2, \qquad (35)$$

where $\gamma = 3(\gamma_1 + \gamma_2 + \gamma_3)$, $\beta_1 \beta = \gamma \beta_2$, and $\kappa = (-3g_3/g_4)^{-\frac{1}{2}}$. It indicates that the gravitino mass (or SUSY breaking mass) strongly depends on the scale of PQ fields as well as Φ_T; for example, for $\mu_\Psi \sim 10^{13}$ GeV and $v_T \sim 10^{11}$ GeV satisfying the SM fermion mass hierarchies [2] one can obtain $m_{3/2} \sim \mathcal{O}(10)$ TeV, and/or subsequently $v_S \sim v_\Theta \sim 10^{11}$ GeV. With the soft SUSY-breaking potential, the radial components of the fields Ψ and $\tilde{\Psi}$ are stabilized at

$$v_\Psi \simeq \mu_\Psi \sqrt{2} \left(\frac{\alpha_2}{\alpha_1} \right)^{1/4}, \qquad v_{\tilde{\Psi}} \simeq \mu_\Psi \sqrt{2} \left(\frac{\alpha_1}{\alpha_2} \right)^{1/4}, \quad (36)$$

respectively.

V. STRING INSPIRED QCD AXIONS

Finally, we consider the four-dimensional effective Lagrangian of the axions, $\theta_i^{\rm st}$ and A_i, and the $U(1)_X$ gauge fields, A_i^μ, which contains the following:

$$K_{T_i \bar{T}_i} \left(\partial^\mu \theta_i^{\rm st} - \frac{\delta_i^{\rm GS}}{16\pi^2} A_i^\mu \right)^2 - \frac{1}{4g_{X_i}^2} F_i^{\mu\nu} F_{i\mu\nu} - g_{X_i} \xi_i^{\rm FI} D_{X_i}$$

$$+ |D_\mu \Phi_i|^2 + \theta_i^{\rm st} {\rm Tr}(G^{\mu\nu} \tilde{G}_{\mu\nu}) + \frac{A_i}{X_i v_i} \frac{\delta_i^{\rm GS}}{16\pi^2} {\rm Tr}(G^{\mu\nu} \tilde{G}_{\mu\nu}),$$

$$(37)$$

where $F_i^{\mu\nu}$ are the $U(1)_{X_i}$ gauge field strengths $F_i^{\mu\nu} = \partial^\mu A_i^\nu - \partial^\nu A_i^\mu$, and the QCD gauge couplings are absorbed into the gluon field strengths. In $|D_\mu \Phi_i|^2$ the scalar fields Φ_i couple to the $U(1)_{X_i}$ gauge bosons, where the gauge couplings g_{X_i} are absorbed into the gauge bosons A_i^μ in the $U(1)_X$ gauge covariant derivative $D^\mu \equiv \partial^\mu + iX_i A_i^\mu$. As mentioned before, the introduction of FI terms $\mathcal{L}_{\rm FI} = -\xi_i^{\rm FI} \int d^2\theta V_{X_i} = -\xi_i^{\rm FI} g_{X_i} D_{X_i}$ leads to the D-term potentials in Eq. (27) where the FI factors $\xi_i^{\rm FI}$ depend on the closed string moduli $\rho_i = \tau_i/2$. The first, third and fourth terms of Eq. (37) stem from expanding the Kahler potential of Eq. (1). Under the anomalous $U(1)_X$ gauge transformation in Eqs. (8) and (9), the first and fifth terms together, and similarly the fourth and sixth terms in Eq. (37), are gauge invariant, that is, the interaction Lagrangians,

$$\mathcal{L}_{X_i}^{\rm int} = A_i^\mu J_\mu^{X_i} - \frac{A_i}{X_i v_i} \frac{\delta_i^{\rm GS}}{16\pi^2} {\rm Tr}(G^{\mu\nu} \tilde{G}_{\mu\nu}),$$

$$\mathcal{L}_{\theta_i^{\rm st}}^{\rm int} = A_i^\mu J_\mu^{X_i} + \theta_i^{\rm st} {\rm Tr}(G^{\mu\nu} \tilde{G}_{\mu\nu}), \qquad (38)$$

are invariant. There are anomalous currents $J_\mu^{X_i}$ coupling to the gauge bosons A_i^μ, that is, $\partial_\mu J_{X_i}^\mu = \frac{\delta_i^{\rm GS}}{16\pi^2} {\rm Tr}(G^{\mu\nu} \tilde{G}_{\mu\nu})$:

$$J_\mu^{X_i} = K_{T_i \bar{T}_i} \frac{\delta_i^{\rm GS}}{8\pi^2} \partial_\mu \theta_i^{\rm st} - iX_i \Phi_i^* \overleftrightarrow{\partial}_\mu \Phi_i + \frac{1}{2} \sum_{\Psi_i} X_i \bar{\Psi}_i \gamma_\mu \gamma_5 \Psi_i.$$

$$(39)$$

Expanding Lagrangian (37) and using $\theta_i^{\rm st} = a_{T_i}/8\pi^2$ it reads

$$\frac{1}{2} (\partial^\mu \tilde{a}_{T_i})^2 + \frac{\tilde{a}_{T_i}}{f_i^{\rm st}} \frac{1}{8\pi^2} {\rm Tr}(G^{\mu\nu} \tilde{G}_{\mu\nu})$$

$$+ \frac{1}{2} (\partial^\mu A_i)^2 + \frac{A_i}{X_i v_i} \frac{\delta_i^{\rm GS}}{16\pi^2} {\rm Tr}(G^{\mu\nu} \tilde{G}_{\mu\nu})$$

$$- J_\mu^{X_i} A_i^\mu + \frac{1}{2g_{X_i}^2} m_{X_i}^2 A_i^\mu A_{i\mu} - \frac{1}{4g_{X_i}^2} F_i^{\mu\nu} F_{i\mu\nu}$$

$$- \frac{g_{X_i}^2}{2} \left(\xi_i^{\rm FI} - \sum_i X_i |\Phi_i|^2 \right)^2, \qquad (40)$$

where $\tilde{a}_{T_i} = f_i^{\rm st} a_{T_i}$ is the canonically normalized Kahler axions with $f_i^{\rm st} = \sqrt{2K_{T_i \bar{T}_i}/(8\pi^2)^2}$. Clearly it indicates that the values of $f_i^{\rm st}$ depend on the Kahler metric and where on the moduli are stabilized. The gauge boson masses obtained by the Higgs mechanism are given by

$$m_{X_i} = g_{X_i} \sqrt{2K_{T_i \bar{T}_i} \left(\frac{\delta_i^{\rm GS}}{16\pi^2} \right)^2 + 2f_{\Phi_i}^2}. \qquad (41)$$

Then the open string axions A_i are linearly mixed with the closed string axions $\tilde{a}_{T_i}$ with decay constants $f_i^{\rm st}$ and $f_{\Phi_i} = X_i v_i$:

$$\tilde{A}_i = \frac{A_i \frac{\delta_i^{\rm GS}}{2} f_i^{\rm st} - \tilde{a}_{T_i} f_{\Phi_i}}{\sqrt{f_{\Phi_i}^2 + (\frac{\delta_i^{\rm GS}}{2} f_i^{\rm st})^2}}, \qquad G_i = \frac{\tilde{a}_{T_i} \frac{\delta_i^{\rm GS}}{2} f_i^{\rm st} + A_i f_{\Phi_i}}{\sqrt{f_{\Phi_i}^2 + (\frac{\delta_i^{\rm GS}}{2} f_i^{\rm st})^2}}.$$

$$(42)$$

Since the $U(1)_X$ is gauged, two linear combinations G_i of the fields A_i and $\tilde{a}_{T_i}$ are eaten by the $U(1)_X$ gauge bosons

and obtain string scale masses, while the other combinations $\tilde{A}_i$ survive to low energies and contribute to the QCD axion. With the given parameters we obtain $m_{X_1} \sim 10^{16}$ GeV and $m_{X_2} \sim 10^{17}$ GeV for $\tau_i/2 \sim 1$, $\delta_1^{GS} = 3$, and $\delta_2^{GS} = 17$. For $f_i^{st} \gg v_i$, the axions $\tilde{A}_i$ as would-be QCD axion are approximated to A_i. Below the scale m_{X_i} the gauge bosons decouple, leaving behind low-energy symmetries which are anomalous global $U(1)_{X_i}$. One linear combination of the global $U(1)_{X_i}$ is broken explicitly by instantons. The rigid example of such would-be QCD axions, and some of its consequences were studied in Ref. [2]. See also Ref. [17].

VI. CONCLUSION

We constructed a string-inspired model as a useful bridge between flavor physics and string theory by introducing two anomalous gauged $U(1)$ symmetries responsible for both the fermion mass hierarchy problem of the SM and the strong CP problem. In the context of supersymmetric moduli stabilization we strongly stabilized the size moduli with positive masses while leaving two axions massless and one axion massive. We showed that, while the massive gauge bosons eat the two axionic degrees of freedom, two axionic directions survive to low energies as the flavored PQ axions.

APPENDIX: NON-ABELIAN DISCRETE SYMMETRY A_4

The group A_4 is the symmetry group of the tetrahedron and the finite groups of the even permutation of four objects having four irreducible representations: its irreducible representations are one triplet **3** and three singlets **1**, **1′**, **1″** with the multiplication rules $\mathbf{3} \otimes \mathbf{3} = \mathbf{3}_s \oplus \mathbf{3}_a \oplus \mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{1}''$, $\mathbf{1}' \otimes \mathbf{1}'' = \mathbf{1}$, $\mathbf{1}' \otimes \mathbf{1}' = \mathbf{1}''$ and $\mathbf{1}'' \otimes \mathbf{1}'' = \mathbf{1}'$. Let (a_1, a_2, a_3) and (b_1, b_2, b_3) denote the basis vectors for two **3**'s. Then, we have

$$
\begin{aligned}
(a \otimes b)_{\mathbf{3}_s} &= \frac{1}{\sqrt{3}}(2a_1 b_1 - a_2 b_3 - a_3 b_2, 2a_3 b_3 - a_2 b_1 \\
&\quad - a_1 b_2, 2a_2 b_2 - a_3 b_1 - a_1 b_3), \\
(a \otimes b_c)_{\mathbf{3}_a} &= i(a_3 b_2 - a_2 b_3, a_2 b_1 - a_1 b_2, a_1 b_3 - a_3 b_1), \\
(a \otimes b)_{\mathbf{1}} &= a_1 b_1 + a_2 b_3 + a_3 b_2, \\
(a \otimes b)_{\mathbf{1}'} &= a_1 b_2 + a_2 b_1 + a_3 b_3, \\
(a \otimes b)_{\mathbf{1}''} &= a_1 b_3 + a_2 b_2 + a_3 b_1.
\end{aligned} \tag{A1}
$$

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